Multicandidate Elections: Aggregate Uncertainty in the Laboratory*

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February 25, 2015

Abstract

The rational-voter model is often criticized on the grounds that two of its central predictions (the *paradox of voting* and *Duverger's Law*) are at odds with reality. Recent theoretical advances suggest that these empirically unsound predictions might be an artifact of an (arguably unrealistic) assumption: the absence of *aggregate uncertainty* about the distribution of preferences in the electorate. In this paper, we propose direct empirical evidence of the effect of aggregate uncertainty in multicandidate elections. Adopting a theory-based experimental approach, we explore whether aggregate uncertainty indeed favors the emergence of non-Duverger's law equilibria in plurality elections. Our experimental results support the main theoretical predictions: sincere voting is a predominant strategy under aggregate uncertainty, whereas without aggregate uncertainty, voters massively coordinate their votes behind one candidate, who wins almost surely.

JEL Classification: C92, D70
Keywords: Rational Voter Model, Multicandidate Elections, Plurality, Aggregate Uncertainty, Experiments

*We are grateful to Guillaume Fréchette, David Myatt and Jacopo Perego, as well as the audience at NYU for their helpful comments.

†Micael Castanheira is a senior research fellow of the Fonds National de la Recherche Scientifique and is grateful for their financial support. This paper was written while he was visiting the NYU economics department, where he benefited from its dynamic intellectual environment.

1 Introduction

Saying that the rational-voter model is not consensual may be an understatement. Voter rationality has been at the center of a heated debate for decades.[1] Its detractors attack this modelling approach on the grounds that some central predictions of the rational voter model are, as summarized by Ledyard (1984, pp7-8), "obviously contradicted by the facts". First, rational-voter models of costly voting highlight the *paradox of voting*: in a large election, "If each person only votes for the purpose of influencing the election outcome, then even a small cost to vote (...) should dissuade anyone from voting. Yet, it seems that many people will put up with long lines, daunting registration requirements and even the threat of physical violence or arrest in order to vote" (Feddersen 2004, p99). Second, rational-voter models of multicandidate elections predict a strong version of *Duverger's Law*: in large plurality elections, all votes should go to the top-two contenders.[2] Instead, Fisher and Myatt (2014, p2) argue that "Duverger's Law (...) sits uncomfortably with the fact that plurality-rule systems generally exhibit multi-candidate support".[3] From these discrepancies, it is tempting to conclude that the rational voter model should be discarded altogether (e.g. Green and Shapiro 1994, Caplan 2007).

However, recent theoretical advances suggest that the empirically unsound predictions of the rational-voter model could be an artifact of a simplifying assumption. It is typically assumed that there is *no aggregate uncertainty* about the distribution of preferences in the electorate. Then, by the law of large numbers, the vote shares of each candidate become known as electorate size grows. As soon as we relax that (unrealistic) assumption, the predictions of the rational-voter model are much more in line with reality. First, this augmented model predicts turnout levels orders of magnitude higher than without aggregate uncertainty (Good and Mayer 1975, Castanheira 2003a, Myatt 2012). Second, stable non-Duverger's Law equilibria (in which three candidates receive a positive fraction of the votes) can be proved to exist in many situations (Myatt 2007, Dewan and Myatt 2007, Bouton and Castanheira 2012, Bouton et al. 2014).

[1] See e.g. Ledyard (1984), Green and Shapiro (1994), Dhillon and Peralta (2002), Feddersen (2004), Degan and Merlo (2009), Kawai and Watanabe (2013), and Ashworth and Bueno de Mesquita (2014).

[2] See, among others, Riker (1982), Palfrey (1989), Myerson and Weber (1993), Cox (1997) and Fey (1997). This literature underlines that, even though they exist, non-Duverger's Law equilibria are typically "expectationally unstable" (Fey 1997), and therefore irrelevant, in that setup.

[3] Recent empirical evidence based on observational data underlines that "Duvergerian forces" do operate in plurality, and lead some (but not all) voters to abandon their most-preferred candidate (Fujiwara 2011, Kawai and Watanabe 2013, Spenkuch 2013, 2014). For evidence based on survey data, see e.g. Blais et al. (2001).

From a theoretical standpoint, aggregate uncertainty alone is thus sufficient to bring the rational voter model much more in line with facts. Yet, competing theories can claim similar achievements (see e.g. Feddersen and Sandroni 2006a,b, Bendor et al. 2011). It is thus fundamental to test empirically whether aggregate uncertainty alone may produce a change in voting behavior that is qualitatively important. This is the main purpose of this paper: we propose direct empirical evidence of the effect of aggregate uncertainty on voting behavior. Our focus is on multicandidate elections under plurality. We adopt a theory-based experimental approach to explore whether aggregate uncertainty indeed favors the emergence of non-Duverger's Law equilibria. And we find that its effects are substantial.

Our main theoretical contribution is to propose a simplified model that captures the effects of aggregate uncertainty in a tractable manner. A fixed number of voters are divided into two groups: a majority and a minority. The majority has two candidates. Each majority voter thus faces the choice of either voting for her preferred candidate (aka voting sincerely) or supporting the other majority candidate (aka voting strategically). Such a *divided majority* setting is ubiquitous in the literature on strategic voting in multicandidate elections.[4]

To understand the theoretical argument, consider first a voter who faces *no aggregate uncertainty*: she knows the parameters of the distribution of preferences in the population. Her only uncertainty is about the actual number of voters who support each candidate. As electorate size grows large, for any voting strategy, she then almost surely knows which candidate will emerge as first, second and third. In this world, her incentive to abandon the third candidate is immense. This is the *psychological effect of Duverger's Law*: "In cases where there are three parties operating under the simple majority single-ballot system the electors soon realize that their votes are wasted if they continue to give them to the third party" (Duverger 1951, p226, cited in Palfrey 1989, p70). Thus, the only stable equilibria are such that all majority voters coordinate their ballots on a same candidate, while the other one receives no vote at all.

Now, what happens if voters expect pre-election polls to be imprecise, *i.e.* if there is aggregate uncertainty? To capture this, we introduce a second state of nature, in which the other majority candidate has stronger support in the population. Then, for some voting strategies, each of the two majority candidates could end up "being third". Should majority voters abandon one of them? We show that voters will want to vote for the majority can-

[4]See, e.g., Palfrey (1989), Myerson and Weber (1993), Cox (1997), Fey (1997), Piketty (2000), Myerson (2002), Dewan and Myatt (2007), Myatt (2007), Bouton and Castanheira (2012), Bouton (2013), Bouton et al. (2014).

didate who wins by the *smallest* margin in her state (technically, this produces the largest pivot probability). The intuition is that they thereby insure themselves against the risk of losing to the minority in the event this candidate turns out to be their best chance to win. This is the “negative feedback loop” identified by Myatt (2007), which operates against the “positive feedback loop” operating in Duverger’s Law. Because of the negative feedback loop, there *also* exists a stable equilibrium in which all three candidates receive a strictly positive vote share. Using Duverger’s words, no candidate is a “wasted ballot”.

Testing the aggregate uncertainty hypothesis in real-world elections is extremely challenging: one would need detailed information on both voter preferences and beliefs (beliefs about aggregate uncertainty and about the other voters’ behavior) that is hard –if not impossible– to obtain from surveys and/or observational data. This is why we propose to test this hypothesis through a controlled laboratory experiment.

We consider two treatments. The only difference between them is that there is no aggregate uncertainty in one (subjects learn the expected distribution of preferences) and there is aggregate uncertainty in the other (subjects do not learn this distribution). Together, the following two pieces of evidence would validate the empirical relevance of the theoretical results: without aggregate uncertainty, subjects should correctly anticipate the expected ranking, and coordinate on the strongest majority candidate. With aggregate uncertainty, they should massively vote sincerely.

Our experimental results provide strong evidence in favor of this joint prediction: the amount of sincere voting under aggregate uncertainty, 63%, is substantially higher than with no aggregate uncertainty, 28%. Conversely, the fraction of votes consistent with the “Duvergerian” strategy of voting for the strongest candidate independently of one’s preference are respectively 32% and 72%. All these differences are statistically significant. These aggregate data nevertheless hide the issue of equilibrium selection, on which theory is silent. In line with theory, all groups select a Duverger’s Law equilibrium under no aggregate uncertainty. Interestingly, they all select the welfare maximizing equilibrium of voting for the candidate with the strongest expected support. In contrast, equilibrium selection turns out to be problematic under aggregate uncertainty. First, not all groups coordinated on the sincere voting equilibrium. Out of 5 groups, only 2 are predominantly “sincere” (93% and 76% of the identified strategies are sincere in these groups). Second, aggregate uncertainty reduces the subjects’ ability to coordinate on a common equilibrium strategy. About one fourth of the votes cannot be attributed to any strategy, against 5% when there is no aggregate uncertainty.

Altogether, we thus observe that aggregate uncertainty does have a first order effect on voting behavior – it reverses the proportions of "sincere" and "strategic" ballots – but also on Condorcet inefficiency.

1.1 Related Literature

Good and Mayer (1975) identified the impact of aggregate uncertainty on pivot probabilities. Myatt (2007, 2012), Dewan and Myatt (2007) and Mandler (2012) show how this influences voting behavior in the rational-voter model. Using a global games approach, Myatt (2007) obtains that Duverger's Law equilibria become fragile, whereas non-Duverger's Law equilibria become the norm. Fisher and Myatt (2001) propose a laboratory experiment to test this model, and find that voters put too little weight on public signals compared to the theoretical predictions. Fisher and Myatt (2014) calibrate their model on survey responses in England and find that it can match voting behavior conditional on aggregate uncertainty being sufficiently high.

By comparison, our simplified model has the advantage of fixing the voters' preference intensity and allowing us to isolate the effect of aggregate uncertainty by varying the relative probabilities of the two states of nature. We find that a non-Duverger's Law equilibrium exists for any probability that is different from 0 or 1. It is thus the presence of aggregate uncertainty, not its level (strong or weak), or the continuum of types and states of nature, or the differential effects of the private and public signals that produces the non-Duverger's Law equilibrium. Moreover, in our setup, the two types of equilibria coexist, which proves empirically relevant. Finally, our experiment complements Fisher and Myatt (2001, 2014): by considering both worlds, with and without aggregate uncertainty, we can clearly isolate its effects on voting behavior.

Outside this literature, aggregate uncertainty has mainly been used in the framework of the Condorcet Jury Theorem, in which voters are ex ante uncertain about their own preference over the candidates (Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1997, Myerson 1998). What motivates three (or more) candidate equilibria in Piketty (2000), Castanheira (2003b), and Bouton and Castanheira (2009, 2012) may thus be mainly due to individual and collective learning effects. In our setup instead, voters have private valued preferences. Information never affects which candidate they prefer, and there are no future periods that may provide an incentive to learn in order to affect future equilibria.

The experimental literature on multicandidate elections is surprisingly small: see Rietz (2008), Laslier (2010) and Palfrey (2012) for detailed reviews of that literature. The seminal paper by Forsythe et al. (1993) is closely related to our paper. They consider three-candidate elections in which a divided majority is opposed to a unified minority. Voters are perfectly informed about the distribution of types in the electorate. They find that, in elections without polls or shared history, plurality rule frequently leads to a victory of the Condorcet loser. However, both polls and shared histories (i) decrease the frequency of such coordination failure among majority voters, and (ii) favor the emergence of Duverger's Law. Forsythe et al. (1996) analyze alternative voting procedures. In Bouton et al. (2014), we propose an experimental setup that is close to the present one, but study a multicandidate Condorcet Jury setup and compare electoral systems. Here, the focus is on first-past-the-post elections, and there is no information aggregation problem.

2 Theoretical Analysis

2.1 The Model

We consider a voting game in which an electorate of fixed and finite size must select a policy P out of three possible alternatives, A, B and C. The electorate is split in two groups: n majority voters, and n_C minority voters. Majority voters have in common that they view C as the worst alternative. Yet, they disagree on which alternative is best: types-t_A prefer A over B whereas types-t_B prefer B over A. In particular, we assume the following utilities:

$$U(P|t_A) = \begin{cases} V > 0 \text{ if } P = A \\ v \in (0, V) \text{ if } P = B \\ 0 \text{ if } P = C, \end{cases} \quad \text{and} \quad U(P|t_B) = \begin{cases} V > 0 \text{ if } P = B \\ v \in (0, V) \text{ if } P = A \\ 0 \text{ if } P = C. \end{cases} \tag{1}$$

For the sake of simplicity, minority voters are assumed to prefer C and be indifferent between A and B; hence their dominant strategy is to vote for C (all the results however extend to the case in which they have a strict preference between A and B). As a consequence, to beat C, either A or B must receive at least n_C ballots. We focus on the interesting case in which C-voters represent a large minority: $n - 1 > n_C > n/2$.[5] The upshot is that, while C is a Condorcet loser, it can win if active voters split their votes between A and B.[6]

[5] When the minority is small, i.e. $n_C < n/2$, alternative C cannot win without support from majority voters and is thus not a real threat.

[6] An alternative interpretation of our setup is that voters vote on whether to reform a status quo policy (C). Two policies could replace this status quo (A and B), and a qualified majority of n_C/n is required for passing a reform (see *e.g.* Dewan and Myatt 2007).

Timing. At **time 0**, nature selects one out of two states of nature $\omega \in \{a, b\}$, with probabilities $q(a)$ and $q(b) = 1 - q(a)$ respectively.

At **time 1**, each voter is assigned a type $t \in T = \{t_A, t_B\}$, by iid draws of a binomial distribution with conditional probabilities $1 > r(t|\omega) > 0$ and $r(t_A|\omega) + r(t_B|\omega) = 1$. These probabilities are common knowledge and vary with the state of nature: $r(t_A|a) > r(t_A|b)$. We shall say that the distribution is *unbiased* if $r(t_A|a) = r(t_B|b)$. Conversely, the distribution is *biased* if $r(t_A|a) \neq r(t_B|b)$. By convention, we focus on the case in which the "more abundant" type is t_A: $r(t_A|a) + r(t_A|b) \geq 1$.

At **time 2,** a *public signal* $s \in S = \{s_0, s_a, s_b\}$ about the state of nature is observed by everyone. We consider two polar scenarios:

1. Under *Aggregate Uncertainty* (AU), the signal is s_0 with probability 1, independently of the state of nature. The public signal is thus uninformative.

2. Under *No Aggregate Uncertainty* (NAU), with probability 1 the signal is s_a (respectively s_b) if the state of nature is a (resp. b). The public signal is then fully informative.

The difference between these two scenarios is at the core of our analysis: we want to study how the voters' information about the expected support for each candidate affects voting equilibria. Intuitively, the two different scenarios distinguish between the case in which, before the election, opinion polls are either so accurate that they leave absolutely no doubt about the expected ranking of the three alternatives, or the polls leave some (possibly minimal) uncertainty about it. In the former case, we have ***no*** *aggregate uncertainty*: voters know the exact parameters of the distribution of types – though not the count of the actual number of voters. If opinion polls can only reveal partial information about this support – in the sense that the expected fraction of voters supporting each alternative remains uncertain – we have *aggregate uncertainty.*[7]

Under AU, and since types are observed only privately, each voter updates her beliefs about the state of nature according to:

$$q(\omega|t, s_0) = \frac{q(\omega)\, r(t|\omega)}{q(a)\, r(t|a) + q(b)\, r(t|b)}. \tag{2}$$

This implies that types t_A put more weight on the state being a than types t_B.

[7] Another way to model these two scenarios would be to make the following assumptions about the states of nature: under *no aggregate uncertainty*, the probability of one of the two states would be zero. Under *aggregate uncertainty*, both states occur with positive probability. The only problem with that modeling strategy is that the two scenarios consider different parameter values.

Under NAU, the public signal trumps the other pieces of information: voters update their beliefs to $q(a|t, s_a) = 1$ and $q(b|t, s_b) = 1$ independently of their type. Finally, the election is held at **time 3**: the alternative with the largest number of votes wins the election – ties are broken by a fair dice – and payoffs are realized.

Strategy space. Each voter may either vote for one of the three alternatives that compete for election or abstain. The action set Ψ is thus:

$$\Psi = \{A, B, C, \varnothing\}.$$

A strategy is a mapping $\sigma : T \times S \to \triangle(\Psi)$, the set of probability distributions over the action set. $\sigma_{t,s}(\psi)$ denotes the probability that a voter with type t and public signal s plays action $\psi \in \Psi$. Note the focus on symmetric strategies (*i.e.* voters with the same type and signal vote in the same way): this reflects the idea that voters are anonymous.

Definition 1 *We can distinguish between* state-contingent strategies *and* non-state-contingent strategies. *The former are such that* $\sigma_{t,s_a}(\psi) \neq \sigma_{t,s_b}(\psi)$. *The latter are such that* $\sigma_{t,s}(\psi) = \sigma_{t,s'}(\psi)\ \forall t, s, s'$.

Quite obviously, *state-contingent strategies* are only possible when there is no aggregate uncertainty. Under aggregate uncertainty, voters do not have sufficient information about the state of nature. Hence, all strategies must be non-state-contingent.

Given a strategy σ, the *expected vote share* of an action ψ in state ω is $\tau_\psi^\omega(\sigma) = \sum_t \sigma_t(\psi) \times r(t|\omega)$. The expected number of ballots ψ cast by active voters is $\mathsf{E}[x(\psi)|\omega, \sigma] = \tau_\psi^\omega(\sigma) \times n$. An *action profile* x is the vector that lists the *realized* number of ballots ψ after the election has occurred.

2.2 Equilibrium Analysis

This section contrasts which equilibria exist and are “robust” in each of the two scenarios under consideration: *Aggregate Uncertainty* (AU) and *No Aggregate Uncertainty* (NAU). We divide this analysis into four parts. The first subsection details necessary preliminaries. The second shows that Two-Party (aka Duverger’s Law) Equilibria always exist. The third proves that a Sincere Voting Equilibrium exists and is robust when there is AU, but is knife-edge or non-existent under NAU. The fourth subsection focuses on large elections and shows that three-candidate equilibria may exist and be stable for any level of AU.

2.2.1 Preliminaries: Payoffs and the Equilibrium Concept

Before starting the equilibrium analysis, note that our setting has been simplified in such a way that voting for C is a dominated action for both majority types t_A and t_B, whereas it is a dominant action for types t_C. The purpose of this simplification is to move quicker through the theoretical analysis. Yet, there will be no doubt that each of the results is robust to the extension in which types t_C have a strict preference between A and B.

What is the trade-off faced by majority-voters? They have to make a decision between voting A or B given their type (t_A or t_B) and the public signal $s \in \{s_0, s_a, s_b\}$. This decision is made without knowing the actual number of voters of each type in the population. They base their decision on the expected value of each action, which depends on *pivot events*: a voter's ballot only affects her utility if it influences the outcome of the election. We denote by piv_{QP} the event that one voter's ballot changes the outcome from a victory of P towards a victory of Q. The probability of piv_{QP} in state $\omega \in \{a, b\}$ is denoted p^{ω}_{QP}.

Since $n_C > n/2$ a ballot can never be pivotal between A and B (see Lemma 2 in Appendix A1).[8] Hence, for a given strategy σ, the value of voting A or B (over abstention) for each type $t \in \{t_A, t_B\}$ and public signal $s \in \{s_0, s_a, s_b\}$ simplifies to:[9]

$$
\begin{aligned}
G(A|t_A, s, \sigma) &= q\,(a|t_A, s)\,Vp^{a}_{AC} + q\,(b|t_A, s)\,Vp^{b}_{AC}, \\
G(B|t_A, s, \sigma) &= q\,(a|t_A, s)\,vp^{a}_{BC} + q\,(b|t_A, s)\,vp^{b}_{BC}, \\
G(A|t_B, s, \sigma) &= q\,(a|t_B, s)\,vp^{a}_{AC} + q\,(b|t_B, s)\,vp^{b}_{AC}, \text{ and} \\
G(B|t_B, s, \sigma) &= q\,(a|t_B, s)\,Vp^{a}_{BC} + q\,(b|t_B, s)\,Vp^{b}_{BC}.
\end{aligned}
$$

In these payoff functions, the pivot probabilities depend on the strategy σ through the expected vote shares τ^{ω}_{P}. Pivot probabilities are continuous in σ and maximized when the expected number of votes of a majority party is equal to n_C. For instance: $\arg\max_{\tau^{\omega}_{A}} p^{\omega}_{AC} = n_C/n$. One can also check that when A's expected vote share is above that of B, the probability of being pivotal in favor of the former is larger than for the latter (see Lemma 1 in Appendix A1). In that case indeed, B has a lower probability of victory and one additional vote for B is less likely to change the outcome.

We are now ready to define our equilibrium concept. We follow Fey (1997), who analyzes the stability of equilibria using a concept initially developed by Palfrey and Rosenthal

[8] The model can however be directly extended to a random number of t_C voters, in which case both p^{ω}_{AB} and the probability of a three-way tie are generally strictly positive.

[9] Note that $\max[G(A|t, s), G(B|t, s)] \geq 0$ for any t and s, and hence that abstention is dominated.

(1991):[10]

Definition 2 (Equilibrium Concept) *A strategy σ^* is an* expectationally stable equilibrium *if $\forall t$ and s, there exists an $\varepsilon > 0$ such that:*

$$(i) \quad \sigma^*_{t,s}(P) > 0 \Rightarrow G(P|t,s,\sigma^*) - G(Q|t,s,\sigma^*) \geq 0,\ \forall Q \in \psi \text{ and}$$

$$(ii) \quad \forall \sigma'_{t,s}(A) \in [\sigma^*_{t,s}(A) - \varepsilon, \sigma^*_{t,s}(A) + \varepsilon] \cap [0,1] \text{ and } \sigma'_{t,s}(B) = 1 - \sigma'_{t,s}(A),$$
$$\sigma'_{t,s}(A) \lesseqgtr \sigma^*_{t,s}(A) \Rightarrow G\left(A|t,s,\sigma'\right) \gtreqless G\left(B|t,s,\sigma'\right).$$

The first element of this definition is a basic best-response requirement: a voter will only cast a P-ballot if it maximizes her expected utility. The intuition for the second element is similar to the concept of stability in Cournot-type competition. Consider the following *tâtonnement* process: let $\sigma^0_{t_A,s}(A)$ be some arbitrary initial strategy in the neighborhood of the equilibrium. For this strategy, a public opinion poll reveals the expected vote shares of each party, which allows voters to compute their best responses. Now, let t_A voters "ϵ-adapt" their strategy, *i.e.* choose a new strategy $\sigma^1_{t_A,s}(A) \in [\sigma^0_{t_A,s}(A) - \epsilon, \sigma^0_{t_A,s}(A) + \epsilon] \cap [0,1]$, where ϵ is positive but arbitrarily small. Then, starting a new iteration: given $\sigma^1_{t_A,s}(A)$, a new poll lets voters calculate their new best response, "ϵ-adapt" their strategy, and so on. The equilibrium is said to be *expectationally stable* if this iteration process produces a sequence $\sigma^k_{t_A,s}(A)$, $k = 1, 2, ...$ that converges to $\sigma^*_{t_A,s}(A)$. Conversely, we exclude equilibria that are not expectationally stable. The reason we focus on this class of equilibria is that stability is essential to identify which equilibria can be expected to be observed in the laboratory (and in real life).

2.2.2 Two-Candidate Equilibria: Duverger's Law

Traditional analyses such as Palfrey (1989) suggest that only *Duverger's Law equilibria* can be expected to arise:

Definition 3 *A* Duverger's Law equilibrium *is an equilibrium in which only two candidates obtain a strictly positive fraction of the votes.*

The intuition for the existence of such equilibria is that, when a majority party's expected vote share is too low, the probability of being pivotal for that party becomes extremely small. To avoid wasting their ballot, majority voters then prefer to coordinate on

[10] The traditional refinement concepts (e.g. trembling-hand perfection and properness) do not have much bite in the context of voting games – see e.g. De Sinopoli (2000).

the other majority party. Our first proposition shows that this logic applies to both AU and NAU, simply because the state of nature becomes irrelevant to determine a party's ranking if its vote share is arbitrarily close to zero anyway:

Proposition 1 *Duverger's Law equilibria always exist and are always expectationally stable, both under Aggregate Uncertainty (AU) and No Aggregate Uncertainty (NAU).*

Proof. See Appendix A2. ■

Yet, the set of feasible Duverger's Law equilibria does depend on the information available to voters: when the signal is s_0, the only two feasible strategies are $\sigma_{t,s}(A) = 1$ and $\sigma_{t,s}(B) = 1$, $t = t_A, t_B$. If instead voters receive a public signal about the state of nature, two other Duverger's Law equilibria can be reached: first, majority voters can coordinate on A when the state is a and on B when the state is b – this is the socially optimal equilibrium. Second, they can coordinate on A when the state is b and on B when the state is a – this is the most socially detrimental Duverger's Law equilibrium. Expectational stability does not help predict which of these equilibria is most likely to be selected.

2.2.3 Three-Candidate Equilibria: Sincere Voting

Duverger's Law equilibria are typically opposed to the "sincere voting equilibrium":

Definition 4 *The* sincere voting equilibrium *is such that, conditional on a public signal, t_A-voters prefer to vote for A, and t_B-voters to vote for B.*

We will say that the sincere equilibrium *exists with probability 1* when it exists for any value of the public signal.

The consensus in the literature is that, for most distributions of preferences in the electorate (implicitly: under NAU), a sincere voting equilibrium *does not* exist (see e.g. Palfrey 1989, Myerson and Weber 1993, and Fey 1997). As shown in Proposition 2 below, the condition behind the existence of the sincere voting equilibrium is that the two majority candidates have (almost) identical vote support.[11] The intuition is the flip-side of the one behind Duverger's Law equilibria: all majority voters would rally behind the strongest of the two majority candidates unless they both have a similar probability of defeating C.

[11] The difference in support that can be sustained in the sincere voting equilibrium crucially depends on the size of the electorate: the larger the electorate, the smaller the admissible proportional difference.

Proposition 2 *Under No Aggregate Uncertainty (NAU),* $\forall n, n_C$, *and a given public signal* $s_\omega \in \{s_a, s_b\}$, *the sincere voting equilibrium only exists if* $|r(t_A|\omega) - 1/2| < \delta(n, n_C)$, *with* $\delta(n, n_C) > 0$. *It is then expectationally stable.*

Proof. See Appendix A3. ■

This condition for existence of the sincere voting equilibrium is obviously quite restrictive. Actually, it is easy to show that in a world with heterogenous preference intensities among majority voters (e.g. v that varies among supporters of a given candidate), this condition is almost never satisfied. Indeed, voters who are close to being indifferent between the two majority candidates will abandon their preferred candidate for any small difference in the support of these two candidates. In such a NAU world, the sincere voting equilibrium is knife edge.

Comparing NAU with AU, we can show that:

Proposition 3 *If the sincere voting equilibrium exists in both states of nature under No Aggregate Uncertainty (NAU), then it also exists under Aggregate Uncertainty (AU).*

Proof. See Appendix A3. ■

The sincere voting equilibrium thus exists with probability 1 as soon as it exists in both states of nature under AU (note still that, from Proposition 2 the sincere equilibrium may only exist in one state of nature). The converse is not true: a sincere equilibrium might exist under AU, and not exist in either state under NAU. Indeed, Proposition 4 shows that the sincere voting equilibrium can exist under AU even when the expected support for the two majority candidates always strongly differs from 1/2:

Proposition 4 *Under Aggregate Uncertainty (AU), for any* $r(t_A|a) = k$, *if* $\frac{q(b)}{q(a)} \in \left(\frac{r(t_B|a)}{r(t_B|b)}, \frac{r(t_A|a)}{r(t_A|b)}\right)$, *there exists a value* $\delta(n, n_C, k) > 0$ *such that for any* $|r(t_B|b) - k| < \delta(n, n_C, k)$ *the sincere voting equilibrium exists. It is then expectationally stable.*

Proof. See Appendix A3. ■

Why is sincere voting more likely to arise under AU? The reason is that a vote may influence the election outcome quite differently in the two states of nature. In particular, a voter may prefer to vote for her most-preferred candidate, say B, even if he is not a serious contender in state a. Such behavior is rational if B can be expected to be a serious contender in state b.[12] This explains why some voters may face ex post regret: when they learn the state of nature after the election, the supporters of one of the two majority candidates will realize that they would have benefitted from voting for the other majority candidate. Such ex post regret is quite frequent. Consider, for instance, Ralph Nader's Florida supporters in the 2000 US presidential election, or left-wing voters in the first-round of the 2002 French Presidential election.

2.2.4 Three-Candidate Equilibria: Beyond Sincere Voting

The above results only offer a very partial characterization of which equilibria may exist: when there is no aggregate uncertainty, only Duverger's Law equilibria are expectationally stable. Conversely, the sincere voting equilibrium is expectationally stable when there is "sufficient" aggregate uncertainty, in the sense that the two states of nature are "sufficiently likely" and "sufficiently symmetric" ($r(t_A|a)$ is not too different from $r(t_B|b)$). These results do not address two issues: (1) What happens when one state of nature is very unlikely? (2) What happens when the two states are not "sufficiently symmetric"?

To address these questions, we focus on the case of large elections (i.e. $n, n_C \to \infty$). This stacks the deck against the existence of the sincere voting equilibrium: since $\delta(n, n_C, k) \to_{n,n_C \to \infty} 0$, the "sufficiently symmetric condition" becomes extremely demanding.

To address question (1), our next proposition shows that, in large elections, essentially any tiny risk of a reversal is sufficient to ensure the existence of a stable equilibrium in the immediate vicinity of sincere voting:

Proposition 5 *Consider a distribution of voters that is unbiased ($r(t_A|a) = r(t_B|b)$) and such that $r(t_A|a) > n_C/n$. Then, for any $q(a) \in (0,1)$, Aggregate Uncertainty (AU) is a necessary and sufficient condition for the existence of an expectationally stable equilibrium arbitrarily close to sincere voting ($\sigma_{t_A,s_0}(A) \simeq \sigma_{t_B,s_0}(B) \simeq 1$) when population size is large.*

Proof. See Appendix A4. ■

This result is in sharp contrast with the traditional analyses that assume away AU. Indeed, an implication of Proposition 5 is that an "almost sincere" equilibrium must exist if there is *any* positive probability that popular support for the two majority candidates can

[12] Interestingly, under AU, heterogenous preference intensities would not hinder the existence of the sincere voting equilibrium. Even voters who are close to indifference between A and B have a strict incentive to vote sincerely.

be reversed. The result that sincere voting is a knife edge equilibrium is only valid when this probability of reversal is exactly zero. In real-life elections, opinion polls must thus be extremely accurate to prevent the existence of the sincere voting equilibrium. In particular, they must completely exclude the possibility that the support for the two majority candidates could be reversed. Arguably, this can only happen when the gap between these two candidates is very high. This rationalizes the empirical evidence about Duverger's Law in Cox (1997, chapter 4): he shows that the voters' propensity to vote sincerely is much higher when the percentage gap between the first and second parties is sufficiently high.

Beyond the above proposition, our next result shows that even if the probability of a *symmetric reversal* ($r(t_A|a) = r(t_B|b)$) is nil, other, non-sincere, three-candidate equilibria exist. A sufficient condition for such equilibria to be stable is that each majority candidate has a positive probability of being a serious contender:

Proposition 6 *Consider a distribution of voters that is biased and such that* $r(t_A|a) > r(t_B|b) > n_C/n$. *Then, for any* $q(a) \in (0,1)$, *Aggregate Uncertainty (AU) is a necessary and sufficient condition for the existence of an expectationally stable equilibrium such that all candidates receive a strictly positive vote share, but voters do not vote sincerely* ($\sigma_{t_A}(A) \in (0,1)$ *and* $\sigma_{t_B}(B) = 1$*).*

Proof. See Appendix A4. ■

Proposition 6 highlights that a symmetric reversal is not necessary to sustain an expectationally stable three-candidate equilibrium. The key condition is that both majority candidates may have larger support than the minority. This condition is sufficient to ensure that, at the equilibrium, A is the expected winner in state a and B is the expected winner in state b. Then, the equilibrium is expectationally stable: suppose that some voters deviate from equilibrium by voting more for A. This implies that C is now a more serious threat in state b. To rebalance risks, these voters now strictly prefer to vote for B. In the words of Myatt (2007), under AU, the three-candidate equilibrium features a *negative feedback*: voters abandoning their most-preferred candidate decrease the incentives of other voters (with the same type) to do so.

This result contrasts with, e.g. Myerson and Weber (1993) and Fey (1997), who consider a world with NAU. In that world, the three-candidate equilibrium requires that the two majority candidates have *exactly* the same expected vote shares. Indeed, since there is no AU, there is no incentive to balance risks across states. The equilibrium thus features a positive feedback: any small deviation triggers a desire by all majority voters to coordinate behind the same majority candidate – it is **never** expectationally stable. For a formal argument, see Proposition 7 in Appendix A5.

3 Experimental Analysis

We ran a series of experiments in order to test our theoretical predictions. Obviously, we must focus here on the results of Propositions 1-4, because the laboratory is not suited to test large election results. Still, as we saw in Propositions 5 and 6, these theoretical results extend – actually in a stronger version – to large electorates.

3.1 Experimental Design and Procedures

We introduced subjects to a game that had the exact same structure as the model of Section 2.1. All participants were given the role of an active voter; passive voters were simulated by the computer.[13] Subjects were allocated to groups of size $n = 12$ and the size of the group of passive voters was $n_C = 7$. The two states of nature were called *blue jar* and *red jar*, whereas the types were called *blue ball* and *red ball*. One of the jars was selected randomly by the computer, with equal probability.

The blue jar contained 2/3 of blue balls and 1/3 of red balls. The red jar contained 2/3 of red balls and 1/3 of blue balls: the distribution was unbiased. One ball was then randomly selected (with replacement) for each participant. After seeing his/her ball, each subject could vote for any one of three candidates: *blue*, *red* or *gray*.[14] *Blue* and *red* were the two majority candidates whereas *gray* was the minority candidate. Subjects were informed that the computer would cast $n_C = 7$ votes for *gray* in each election.

The subjects' payoff depended on their type and on the election winner. If the color of the winner matched that a given participant's ball (type), this participant had a payoff of €2. If the winner was blue or red but did not match the color of the subject's ball, his/her payoff was €1.1. Finally, if gray won, all subjects' payoff was €0.20.

We ran two treatments: in the *Aggregate Uncertainty* treatment (AU) the selected jar

[13]Morton and Tyran (2012) show that preferences in one group are not affected by preferences of an opposite group. Therefore, having computerized rather than human subjects should not alter the behavior of majority voters in a significant way. In Battaglini et al. (2008, 2010), partisans equivalent to our passive voters are also simulated by the computer, and Bouton et al. (2014) follow a similar experimental strategy.

[14]As in Guarnaschelli et al. (2000), abstention was not allowed (remember that abstention is a weakly dominated action in our setup). In a setting related to ours, Forsythe et al. (1993) allowed for abstention and found that the abstention rate was as low as 0.65%.

was **not** revealed before voting, whereas it was in the *No Aggregate Uncertainty* treatment (NAU). Everything else was held constant across the two treatments.

Experiments were conducted at the Experimental Economics Laboratory at the University of Valencia (LINEEX) in November 2014. We ran one session for each treatment, with 60 subjects (or 5 independent groups) each. No subject participated in more than one session. Students interacted through computer terminals, and the experiment was programmed and conducted with *z-Tree* (Fischbacher 2007). All experimental sessions were organized along the same procedure: subjects received detailed written instructions (see Appendix A6), which an instructor read aloud. Before starting the experiment, subjects were asked to answer a questionnaire to check their full understanding of the experimental design. Right after that, they played one of the treatments for 80 periods in fixed groups.[15] At the end of each period, subjects were given the following information: (i) which was the selected jar; (ii) which color won the election, (iii) the number of votes for each alternative and (iv) their payoff for that period. To determine payment at the end of the experiment, the computer randomly selected eight periods and participants earned the total of the amount earned in these periods. In total, subjects earned an average of €15.22, including a show-up fee of €4. Each session lasted approximately 75 minutes.

3.2 Equilibria and Hypotheses

In this section, we summarize the results of Propositions 1-4 and use them to formulate testable hypotheses. Section 2 identified two types of equilibria: Duverger's Law vs. Sincere Voting. Under the parameters chosen for the experimental setting, Sincere Voting is only an equilibrium in the AU treatment. Which of the various Duverger's Law equilibria can materialize also depends on the treatment: there are two such equilibria in AU. One in which all majority voters coordinate on voting blue, and the other one in which they coordinate on red. Two additional equilibria in state-contingent strategies can emerge in treatment NAU: voting for the color of the selected jar (*voting the jar* for short) and voting for the majority color opposite to that of the selected jar (*voting opposite* for short). Table 1 summarizes these results.

[15] We chose to have fixed matching for two reasons. First, fixed matching creates a shared history that facilitates subjects' learning about the strategies played by other subjects, and hence coordination. For instance, Forsythe et al. (1993, 1996) observe that Duverger's Law equilibria emerge more easily among voters with a common history – see also Rietz 2008. Thus, by making this choice, we stack the deck against the presence of sincere voting equilibria in treatment NAU. Second, having fixed matching allows for comparability with the results Bouton et al. (2014), which analyzes a similar setup but with common values.

Type of Equilibrium		**AU**	**NAU**
Sincere Voting		✓	×
Duverger's Law	Blue	✓	✓
	Red	✓	✓
	Voting the jar	–	✓
	Voting opposite	–	✓

Table 1: Set of equilibria in each treatment.

The main testable implication for our theoretical results is thus that:

Hypothesis 1 *The frequency of sincere voting will be (weakly) higher in treatment AU than in treatment NAU.*

Beyond this hypothesis, and despite the fact that our theory is silent about equilibrium selection, we can make an informed conjecture about which of these equilibria are most likely to arise. Bouton et al. (2014) shows that, even with common values, Duverger's Law equilibria are slow to emerge – when they emerge at all – in symmetric environments. We thus expect sincere voting to be a focal strategy in treatment AU:

Hypothesis 2 *In treatment AU, voters will converge to the sincere voting equilibrium.*

By contrast, sincere voting is *not* an equilibrium in treatment NAU. But, which of the four possible Duverger's Law equilibria will voters select? Since the distribution of preferences is known, an obvious "strong contender" emerges. This should produce a strong incentive to vote for that candidate (see Section 2.2.2). We thus expect voters to coordinate on the Pareto dominant equilibrium:

Hypothesis 3 *Groups in treatment NAU will converge to the state-contingent Duverger's Law equilibrium and vote the jar.*

3.3 Experimental Results

We first detail the results for the AU and NAU treatments respectively. Then, in Section 3.3.3, we put these results in perspective by comparing them and we test our main hypothesis (i.e. Hypothesis 1). Unless stated otherwise, the results use all the data of the experiment. Still, all the results are robust to only considering the last 40 or 20 periods.

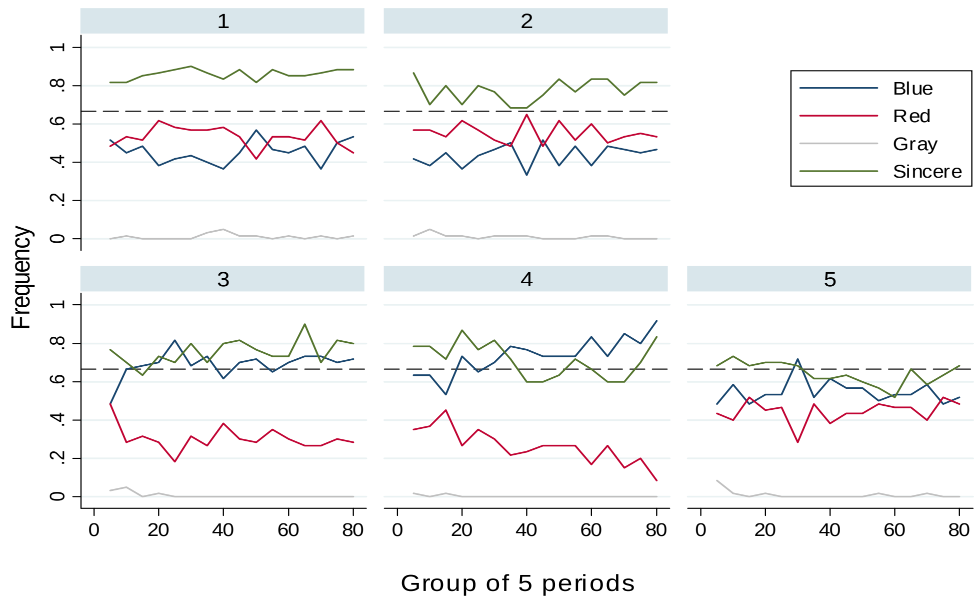

Figure 1: Aggregate behavior in each group in treatment AU. The blue, red, and gray lines display the aggregate frequency of voting that color. The green line represents the frequency of sincere voting. The dashed black line represents the number of votes required to defeat the Condorcet loser.

3.3.1 Aggregate Uncertainty (AU)

Figure 1 displays aggregate behavior in treatment AU, separated by group. In particular, the blue, red and gray lines correspond to the frequency of voting blue, red or gray by blocks of 5 periods. In addition, we computed the fraction of the votes that are "sincere" (a red or blue vote when the voter respectively received a red or blue ball). This fraction is displayed by the green line.[16] Finally, the dashed horizontal line represents the number of votes required to defeat the Condorcet loser.

Consider first the subjects' propensity to vote for each alternative. Based on that information, we first observe that almost no vote goes to gray. Second, we can contrast two types of dynamics among these groups: groups 1, 2 and 5 continuously display split support for blue and red. In these groups, the frequencies of voting blue and red hover around 50%

[16]Clearly, while the frequencies of voting blue, red and gray must sum to 1, the frequency of voting sincerely only sums to one with the probability of voting against one's color (not displayed).

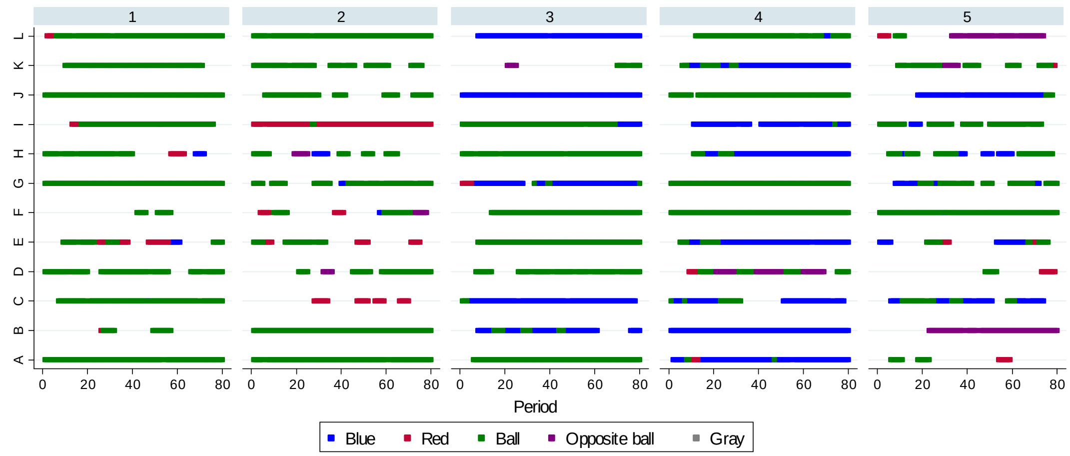

Figure 2: Individual Behavior in groups in treatment AU. Each panel corresponds to a different group. A, B, C... to L in the vertical axis refer to each of the 12 players in each group. Each line plots the strategies identified throughout the 80 periods.

throughout the entire experiment.[17] By contrast, the propensity to vote red declines over time in groups 3 and 4, suggesting progressive convergence to the blue Duverger's Law equilibrium.

The evidence of sincere voting is different – actually much more mixed – when we control for subject type. This allows us to identify which of the red and blue votes are actually "sincere". Figure 1 shows that subjects' propensity to vote sincerely remains above 66% in groups 1, 2 and 3, whereas it is slightly below in group 4 and is lowest in group 5.[18] This contrasts with the picture that emerged when we ignored subject type and only focused on the shares of blue and red votes: groups 3 and 4 seemed to converge to the blue Duverger's Law equilibrium,[19] whereas vote shares in group 5 seemed consistent with sincere voting. This shows why it is crucial to control for voter preferences when bringing the model to the data. Yet this mixed evidence also cries for more detail about the subjects' individual strategies to identify whether an equilibrium is reached – and which one.

[17] These frequencies are 45.41% and 53.43% in group 1; 43.64% and 55.21% in group 2; and 54.68% and 44.38% in group 5.

[18] The fraction of sincere ballots was 85.94%, 77.50%, 75.63%, 71.25%, and 64.37% in groups 1-5 respectively.

[19] The levels of sincere voting is unusually high in group 3 given the high frequency of voting for blue. This is because the realized frequency of the blue jar being selected was higher than its expected value: 57.50%.

Our experimental data, which provides a sequence of 80 choices for each subject allow us to do that. We propose the following identification procedure to identify the subjects' actual strategy: first, we identify a set of 7 plausible pure strategies: (1) voting for blue, (2) voting for red, (3) voting for gray, (4) voting sincerely, (5) voting opposite to sincerely (that is, for the majority color that does not coincide with the ball), (6) voting the jar and (7) voting opposite to the jar (the last two strategies are only relevant in treatment NAU, see below). Second, we attribute a particular strategy to a subject in a particular period if such strategy was played for at least five consecutive periods. This will uniquely identify a strategy in most cases. When a subject's actions are compatible with more strategies, we attribute the compatible strategy that he/she played the most across the 80 periods of the experiment.

The number of plays compatible with one of these pure strategies is as high as 75.77%. Figure 2 displays these identified strategies: each panel corresponds to an independent group and each line to a particular subject in that group. One remarkable feature is the predominance of green, which reveals the large number of subjects who play the sincere voting strategy. The five groups are actually ordered by decreasing number of voters who play that strategy for most periods: the number of voters for which we identify the sincere voting strategy in at least 40 periods is 8, 6, 6, 4 and 3 in groups 1 to 5 respectively. The number of subjects who play (for most periods) a pure strategy of voting either blue or red independently of their types is concomitantly increasing from groups 1 to 4: no subject adopts such a strategy in group 1; one subject plays red in group 2; and 5 and 7 subjects play blue in groups 3 and 4. Interestingly, convergence to a Duverger's Law equilibrium remained only partial in groups 3 and 4: even though red *never* won any election in the last 40 periods, respectively 5 and 4 subjects kept voting sincerely. Group 5 displays another type of coordination failure: first, we cannot identify as many strategies as in the other groups. Second, two subjects vote opposite (a dominated strategy), two vote sincerely and 1 votes blue.

In light of these results, we conclude that Hypothesis 2 must be rejected: while the modal strategy is sincere voting (among all identified strategies, 63.32% can be attributed to sincere voting), it is only overwhelming in groups 1 and 2. The subjects are split between those trying to reach the sincere voting equilibrium and the blue Duverger's Law equilibrium in groups 3 and 4 – the number of subjects playing blue is however sufficient in these groups to make blue win respectively 80% and 97.5% of the times in the last 40 periods. Finally, group 5 does not display any convergence to an equilibrium: gray is the modal winner, with 45% of victories in the last 40 periods.

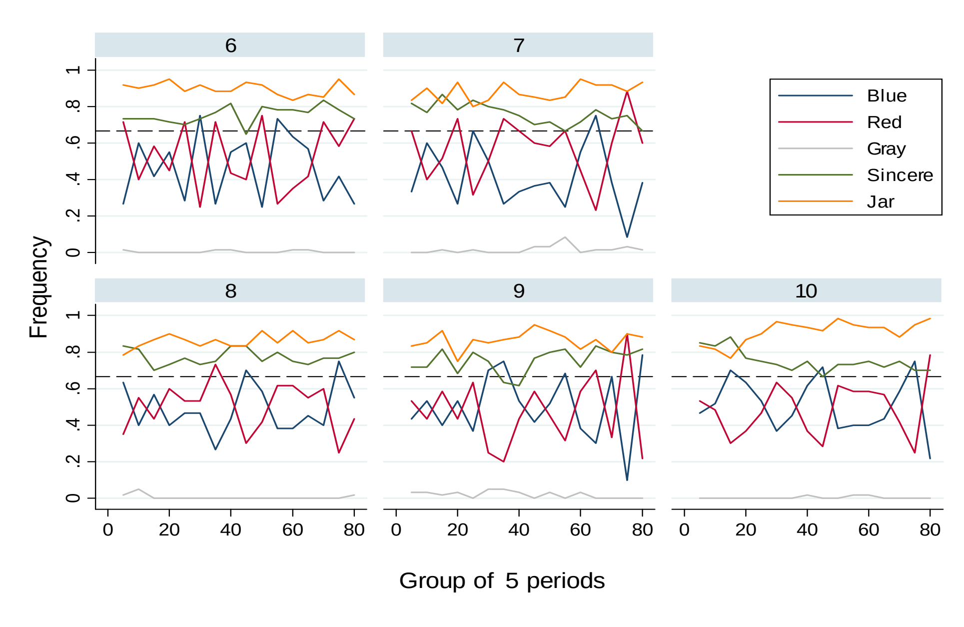

Figure 3: Aggregate behavior in each group in treatment NAU. The blue, red, and gray lines display the aggregate frequency of voting that color. The green line represents the frequency of sincere voting, and the yellow one represents the frequency of voting for the color of the selected jar. The dashed black line represents the number of votes required to defeat the Condorcet loser.

3.3.2 No Aggregate Uncertainty (NAU)

Now, let us focus on the NAU treatment, in which subjects were given a perfectly informative signal about the color of the jar. Figure 3 displays aggregate behavior in each group in treatment NAU. On top of the lines represented in Figure 1, we add the orange line which corresponds to the frequency of voting the jar. Figure 3 shows a clear and common pattern across all groups. Every single group converges to this state-contingent Duverger's Law equilibrium. The overall frequency of votes for the jar was 88.22%, with little heterogeneity across groups (89.58%, 87.81%, 86.25%, 86.46% and 91.04% in groups 6 to 10, respectively). Interestingly, this is still compatible with a high frequency of sincere voting. Indeed, in expectation, 2/3 of the subjects receive a ball that matches the color of the selected jar. To tell these two strategies apart, consider the behavior of those subjects whose ball did not match: their overall frequency of voting the jar was 67.33% while the frequency of sincere voting was 30.24%.

Figure 4 displays individual strategies following the identification procedure described in

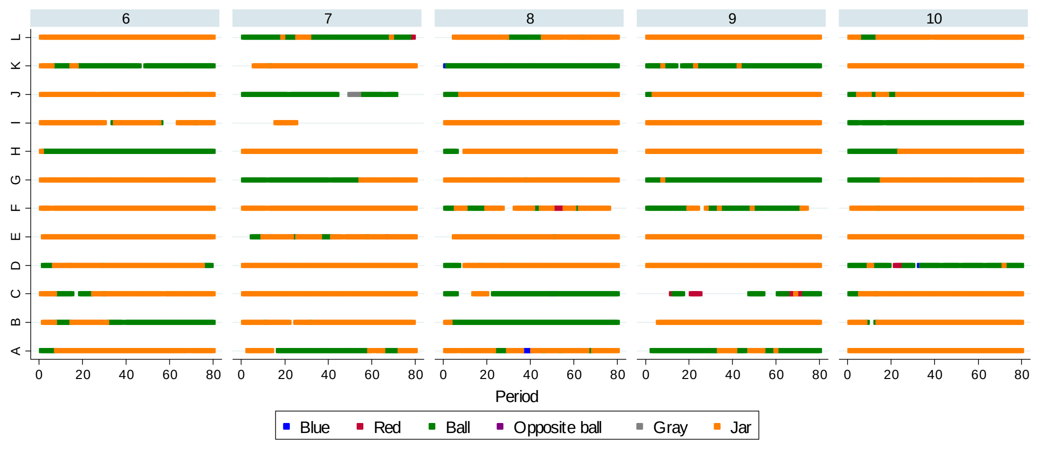

Figure 4: Individual Behavior in groups in treatment NAU. Each panel corresponds to a different group. A, B, C... to L in the vertical axis refer to each of the 12 players in each group. Each line plots the strategies identified throughout the 80 periods.

the previous subsection. Like in Figure 2, each panel corresponds to an independent group and each line to a particular subject in that group. Interesting patterns stand out. First, the number of actions compatible with one of the aforementioned pure strategies is very high (94.96%). Second, the strategy of voting the jar clearly stands out: it represents 71% of the identified strategies. 58.33% of the subjects played it at least 90% of the time and 23.33% of the subjects played it 100% of the time. Only a small minority of the subjects vote sincerely: no subject played that strategy 100% of the time and only 11.67% at least 90% of the time. We do not find evidence of any other strategy being played consistently. As a consequence, the color of the selected jar won 99% of the elections. This provides strong support for Hypothesis 3.

3.3.3 Comparison

In this section we compare behavior across different treatments and test Hypothesis 1. As we saw in Section 3.3.1, focusing on the percentage of actions consistent with sincere voting can give spurious information. The right approach is to exploit the identified strategies. Table 2 summarizes the frequencies of the identified strategies in each treatment. We observe a clear difference across treatments: among the identified strategies, the percentages of sincere voting in treatments AU and NAU are 63.32% and 28.15% respectively. This difference is

	Treatment AU	Treatment NAU
Blue	26.81	0.15
Red	5.58	0.59
Gray	0.00	0.11
Sincere	63.32	28.15
Opposite	4.29	0.00
Selected Jar	–	71.00
% Identifies strategies	75.77	94.96

Table 2: Percentage of identified strategies in each treatment.

statistically significant (Mann-Whitney, $z = 2.611$, $p = 0.009$). We can therefore reject the null hypothesis that the amount of sincere voting is the same across treatments, and validate Hypothesis 1. Actually, this comparison holds for every single independent group: the frequency of sincere voting is at least 44.10% in every group in treatment AU whereas it is at most 31.21% in treatment NAU. This can be seen in Table 3, which disaggregates identified strategies by independent group.

Next, we compare the strategies consistent with Duverger's Law equilibria. In treatment AU, the strategies consistent with Duverger's Equilibria are voting blue or voting red. Two more strategies are available in treatment NAU: voting for or against the selected jar. The percentage of strategies consistent with any Duverger's Law equilibrium are respectively 71.74% and 32.39% in treatments NAU and AU.[20] This difference is statistically significant (Mann-Whitney, $z = 2.611$, $p = 0.009$). Here again, the difference holds for every group: even group 4, which has the largest fraction of subject/periods playing blue in treatment AU, is below group 8, which has the lowest percentage of strategies consistent with a Duverger's Law equilibrium in treatment NAU. This result is not an artifact of the slow convergence observed in treatment AU: the same results hold when we restrict our attention to the last 40 or 20 periods.

Another interesting difference between the two treatments is the overall percentage of identified strategies, which is 75.77% in treatment AU and 94.96% in treatment NAU. This difference is statistically significant (Mann-Whitney, $z = 2.402$, $p = 0.016$). In our view this difference is evidence of the strategic uncertainty observed in treatment AU. This results from the tension between sincere voting and Duverger's Law (see page 19 and the Conclusions). Note that, under no aggregate uncertainty, Forsythe et al. (1993) obtain

[20] Note that this result is robust to restricting the attention to the state-contingent Duverger's Law equilibrium under treatment NAU (against all other Duverger's Law equilibria in treatment AU). Hence, this comparison is not a mere artifact of having more equilibria in treatment AU.

Treatment	Group	Identified strategy Blue	Red	Gray	Sinc	Opp	Jar	% Identified Strategies
AU	1	1.71	5.12	0.00	**93.17**	0.00	-	73.23
	2	1.83	19.08	0.00	**76.34**	2.75	-	68.23
	3	43.37	0.62	0.00	**55.39**	0.62	-	84.06
	4	**51.32**	1.03	0.00	44.10	3.55	-	90.94
	5	25.54	4.67	0.00	**52.75**	17.03	-	62.40
NAU	6	0.00	0.11	0.00	25.53	0.00	**74.36**	97.50
	7	0.00	0.59	0.59	28.52	0.00	**70.31**	88.75
	8	0.65	0.54	0.00	31.21	0.00	**67.60**	96.46
	9	0.00	1.33	0.00	33.70	0.00	**64.96**	93.65
	10	0.11	0.42	0.00	22.12	0.00	**77.35**	98.44

Table 3: Percentage of identified strategies in each matching group. Modal strategy in each group is indicated in bold.

convergence to Duverger's Law equilibria despite the absence of an explicit "public signal" such as the color of the jar: the sharing of a common history is sufficient to make focal a candidate who fared well in past elections.[21] Similarly, in a Condorcet Jury setup, Bouton et al. (2014) observe that either sincere voting or a Duverger's Law equilibrium emerges, depending on the strength of the minority candidate.

4 Conclusions

We used a theory-based experimental approach to test the effects of aggregate uncertainty on voting behavior in plurality elections. The main theoretical prediction is that voters should coordinate on only two candidates when there is no aggregate uncertainty, whereas they can also coordinate on a three-candidate equilibrium under aggregate uncertainty. The experimental results show that the quantitative impact of aggregate uncertainty is first order. While only 24% of the subjects vote sincerely under no aggregate uncertainty, 61% do so when there is aggregate uncertainty. The fact that voters do behave according to Duverger's Law in one case, and sincerely in the other exposes the claim that the rational voter model lacks empirical relevance. It suggests instead that it may just be an artifact of the classical – technically convenient – *no aggregate uncertainty* assumption.

The experiment also proved valuable to identify issues on which theory is largely silent:

[21] It is well known that strong focal points ease coordination. See, for instance, Metha et al. (1994) or Fehr et al. (2011) and references therein.

the nature of the voters' coordination problem appears to be qualitatively different with and without aggregate uncertainty. Only Duverger's Law equilibria exist in the absence of aggregate uncertainty. Then, the coordination problem boils down to identifying one candidate all majority voters should vote for. The supporters of a candidate expected to rank third should desert him; this *positive feedback loop* boosts the top two contenders. With aggregate uncertainty, Duverger's Law equilibria coexist with a non-Duverger's Law equilibrium. In the latter equilibrium, a candidate expected to be "at risk" may receive *more* support from the electorate; this is Myatt (2007)'s *negative feedback loop*, which preserves the support of the third contender. Our experimental results suggest that voters may fail to agree on which of these two feedback loops should dominate their voting behavior, which increases the occurrence of coordination failures. Such failures are costly, since they allow the minority candidate (a Condorcet loser) to win more often than necessary – realized payoffs stood 21% below the ones achievable in the absence of coordination failure. These results thus suggest that coordination failures may contribute strongly to discontent with the plurality voting system. Future research should shed more light on the circumstances that could facilitate coordination when there is aggregate uncertainty.

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Appendices

Appendix A1: Pivot probabilities and preliminary proofs

First, note that a vote, say for A, can only be pivotal against C if it either breaks a tie ($x_A = x_C$ without that vote) or makes a tie ($x_A = x_C - 1$ without that vote). Given the binomial distribution of t_A and t_B voters and their strategies σ_{t_A} and σ_{t_B} that determine the expected vote shares τ_A and τ_B. When voters play undominated strategies, we have:

$$\begin{aligned}
\tau_A^\omega(\sigma, s) &= r(t_A|\omega)\,\sigma(A|t_A, s) + r(t_B|\omega)\,\sigma(A|t_B, s),\\
\tau_B^\omega(\sigma, s) &= r(t_A|\omega)\,(1-\sigma(A|t_A, s)) + r(t_B|\omega)\,(1-\sigma(A|t_B, s)),
\end{aligned}$$

and C receives exactly n_C votes. The pivot probabilities are given by:

$$p_{AC}^\omega \equiv \Pr(piv_{AC}|\omega) = \frac{(n-1)!}{2}\frac{(\tau_A^\omega)^{n_C-1}(\tau_B^\omega)^{n-n_C-1}}{(n_C-1)!(n-n_C-1)!}\left[\frac{\tau_A^\omega}{n_C} + \frac{\tau_B^\omega}{n-n_C}\right], \tag{3}$$

$$p_{BC}^\omega \equiv \Pr(piv_{BC}|\omega) = \frac{(n-1)!}{2}\frac{(\tau_B^\omega)^{n_C-1}(\tau_A^\omega)^{n-n_C-1}}{(n_C-1)!(n-n_C-1)!}\left[\frac{\tau_B^\omega}{n_C} + \frac{\tau_A^\omega}{n-n_C}\right], \tag{4}$$

where the two terms between brackets represent the cases in which one vote respectively breaks and makes a tie. Note that pivot probabilities are continuous in τ_A^ω and τ_B^ω.

To characterize equilibrium, it will prove useful to characterize the payoff difference between actions A and B. Dropping the conditioning on σ for the sake of brevity, we have:

$$G(A|t_A, s) - G(B|t_A, s) = q(a|t_A, s)\ [Vp_{AC}^a - vp_{BC}^a] + q(b|t_A, s)\ [Vp_{AC}^b - vp_{BC}^b], \tag{5}$$

$$G(A|t_B, s) - G(B|t_B, s) = q(a|t_B, s)\ [vp_{AC}^a - Vp_{BC}^a] + q(b|t_B, s)\ [vp_{AC}^b - Vp_{BC}^b]. \tag{6}$$

It is straightforward to check that:

$$G(A|t_A, s) - G(B|t_A, s) \geq G(A|t_B, s) - G(B|t_B, s). \tag{7}$$

We are now in a position to characterize the following three lemmas:

Lemma 1 *The pivot probability ratio $p_{AC}^\omega/p_{BC}^\omega$ is strictly larger than 1 and increasing to infinity for $n_C/n = k$ and $n \to \infty$ if and only if $\tau_A^\omega > \tau_B^\omega$.*

Proof. *Consider the case in which $\tau_A^\omega > \tau_B^\omega$. From (3) and (4), we have:*

$$\frac{p_{AC}^\omega}{p_{BC}^\omega} = \frac{(\tau_A^\omega)^{n_C-1}(1-\tau_A^\omega)^{n-n_C-1}}{(1-\tau_A^\omega)^{n_C-1}(\tau_A^\omega)^{n-n_C-1}} \frac{\frac{\tau_A^\omega}{n_C}+\frac{1-\tau_A^\omega}{n-n_C}}{\frac{1-\tau_A^\omega}{n_C}+\frac{\tau_A^\omega}{n-n_C}} = \left(\frac{\tau_A^\omega}{1-\tau_A^\omega}\right)^{2n_C-n} \frac{\frac{\tau_A^\omega}{1-\tau_A^\omega}+\frac{n_C}{n-n_C}}{1+\frac{\tau_A^\omega}{1-\tau_A^\omega}\frac{n_C}{n-n_C}}.$$

Noting that this pivot probability ratio is equal to 1 in $\tau_A^\omega = 1/2$ and that it is strictly increasing in τ_A^ω proves the Lemma. ■

Lemma 2 $p_{AB}^\omega = 0$.

Proof. *Given that $n_C > n/2$, if the vote count for A and for B are equal, it must be that both are strictly below n_C, in which case, neither A nor B may win.* ■

Lemma 3 *If an equilibrium σ^* is* strict, *i.e. all voters have strict best response, then σ^* is necessarily expectationally stable. The converse is not true.*

Proof. In a strict equilibrium, $\forall t, s$, we must have either $G(A|t,s) > G(B|t,s)$ or $G(A|t,s) < G(B|t,s)$. Now, consider any P such that $\exists t, s$ such that $\sigma_{t,s}(P) = 1$. By the continuity of $G(P|t,s)$ in the τs, we have that $\exists \varepsilon > 0$ such that if $\sigma_{t,s}(A) > 1-\varepsilon$, then $G(P|t,s) > G(P'|t,s)$ with $P' \neq P \in \{A, B\}$. ■

Appendix A2: Proof of Section 2.2.2

Proof of Proposition 1. Consider e.g. $\sigma_{t_A,s}(A) = \varepsilon$ and $\sigma_{t_B,s}(B) = 1$. From (3) and (4), we have:

$$\frac{p_{AC}^\omega}{p_{BC}^\omega} = \left(\frac{\tau_A^\omega}{\tau_B^\omega}\right)^{2n_C-n} \frac{\tau_A^\omega(n-n_C)+\tau_B^\omega n_C}{\tau_A^\omega n_C + \tau_B^\omega(n-n_C)} \underset{\varepsilon\to 0}{\to} 0.$$

Hence, from (5) and (6), there exists a non-empty set $(0, K)$ such that $G(A|t,s) - G(B|t,s) < 0$, $\forall t, s$ whenever ε is in this set. The equilibrium being strict, expectational stability follows immediately (Lemma 3). ■

Appendix A3: Proofs of Section 2.2.3

Proof of Proposition 2. We focus on the case $s_\omega = s_a$ and start with $r(t_A|a) = r(t_B|a)$. Under sincere voting, $\sigma_{t_A,s_a}(A) = 1 = \sigma_{t_B,s_a}(B)$, (3) and (4) imply $p_{AC}^a = p_{BC}^a$. Thus, from (5), we have:

$$\begin{aligned} G(A|t_A,s_a) - G(B|t_A,s_a) &= Vp_{AC}^a - vp_{BC}^a \\ &= p_{AC}^a[V-v] > 0. \end{aligned}$$

Similarly, from (6), we have that:

$$\begin{aligned} G(A|t_B, s_a) - G(B|t_B, s_a) &= vp^a_{AC} - Vp^a_{BC}, \\ &= p^a_{AC}\,[v - V] < 0 \end{aligned}$$

Sincere voting is thus an equilibrium strategy. Next, by the continuity of pivot probabilities with respect to τ^ω_A and τ^ω_B, there must exist a value $\delta(n, n_C) > 0$ such that sincere voting is an equilibrium for any $|r(t_A|a) - r(t_B|a)| < \delta(n, n_C)$.

Expectational stability directly follows from the fact that the equilibrium is "strict" (Lemma 3).

■

Proof of Proposition 3. The proof is straightforward. For the sincere voting equilibrium to exist in state ω under NAU, we must have:

$$G(A|t_A, s) - G(B|t_A, s) \geq 0 \geq G(A|t_B, s) - G(B|t_B, s). \tag{8}$$

Now, note that, under AU, we have that

$$G(A|t, s_0) - G(B|t, s_0) \equiv q(a|t, s_0)\ \ [G(A|t, s_a) - G(B|t, s_a)] + q(b|t, s_0)\ \ [G(A|t, s_b) - G(B|t, s_b)]$$

Thus, if condition (8) is satisfied in both states of nature, we must have that

$$G(A|t_A, s_0) - G(B|t_A, s_0) \geq 0 \geq G(A|t_B, s_0) - G(B|t_B, s_0).$$

■

Proof of Proposition 4. First, note that

$$q(a|t_A) > q(b|t_A) \text{ and } q(a|t_B) < q(b|t_B),$$

if and only if $\frac{r(t_A|a)}{r(t_A|b)} > \frac{q(b)}{q(a)} > \frac{r(t_B|a)}{r(t_B|b)}$.

Second, consider the unbiased case, *i.e.* $r(t_A|a) = r(t_B|b)$. Under sincere voting, $\sigma_{t_A,s_0}(A) = 1 = \sigma_{t_B}(B)$, (3) and (4) imply $p^a_{AC} = p^b_{BC} > p^b_{AC} = p^a_{BC}$. Then, from (5), we have that (we drop the notation s_0 to ease readability):

$$\begin{aligned} G(A|t_A) - G(B|t_A) &= q(a|t_A)\ \ [Vp^a_{AC} - vp^a_{BC}] + q(b|t_A)\ \ [Vp^b_{AC} - vp^b_{BC}], \\ &= q(a|t_A)\ \ [Vp^a_{AC} - vp^b_{AC}] + q(b|t_A)\ \ [Vp^b_{AC} - vp^a_{AC}], \\ &= V\left(p^a_{AC}q(a|t_A) + p^b_{AC}q(b|t_A)\right) - v\left(p^a_{AC}q(b|t_A) + p^b_{AC}q(a|t_A)\right), \\ &> v\left(p^a_{AC} - p^b_{AC}\right)(q(a|t_A) - q(b|t_A)) > 0 \end{aligned}$$

The first inequality comes from equating $V = v$. Similarly, from (6), we have that:

$$\begin{aligned} G(A|t_B) - G(B|t_B) &= q(a|t_B)\ [vp^a_{AC} - Vp^a_{BC}] + q(b|t_B)\ [vp^b_{AC} - Vp^b_{BC}], \\ &= q(a|t_B)\ [vp^a_{AC} - Vp^b_{AC}] + q(b|t_B)\ [vp^b_{AC} - Vp^a_{AC}], \\ &= v\left(p^a_{AC} q(a|t_B) + q(b|t_B) p^b_{AC}\right) - V\left(p^b_{AC} q(a|t_B) + p^a_{AC} q(b|t_B)\right) \\ &< V\left(p^a_{AC} - p^b_{AC}\right)\left(q(a|t_B) - q(b|t_B)\right) < 0. \end{aligned}$$

Again, the first inequality comes from equating $V = v$. Sincere voting is thus an equilibrium strategy.

Third, by the continuity of pivot probabilities with respect to τ^ω_A and τ^ω_B, it immediately follows that there must exist a value $\delta(n, n_C) > 0$ such that sincere voting is an equilibrium for any $|r(t_A|a) - r(t_B|b)| < \delta(n, n_C)$.

Finally, the proof of expectational stability directly follows from the fact that the equilibrium is "strict" (Lemma 3). ■

Appendix A4: Proofs of Section 2.2.4

Proof of Proposition 5. Proposition 4 already shows that a sincere equilibrium exists when $q(a|t)$ is sufficiently close to 1/2. It remains to prove existence of an "almost sincere" equilibrium when $q(a|t)$ is outside this range. Note that, in such an equilibrium, we must have:

$$G(A|t_A, s_0) - G(B|t_A, s_0) = q(a|t_A, s_0)\ [Vp^a_{AC} - vp^a_{BC}] + q(b|t_A, s_0)\ [Vp^b_{AC} - vp^b_{BC}] \geq 0, \quad (9)$$
$$G(A|t_B, s_0) - G(B|t_B, s_0) = q(a|t_B, s_0)\ [vp^a_{AC} - Vp^a_{BC}] + q(b|t_B, s_0)\ [vp^b_{AC} - Vp^b_{BC}] \leq 0. \quad (10)$$

These are the same conditions as:

$$\begin{aligned} \frac{q(a|t_A, s_0)}{q(b|t_A, s_0)}\ [Vp^a_{AC} - vp^a_{BC}] &\geq vp^b_{BC} - Vp^b_{AC} \\ \frac{q(a|t_B, s_0)}{q(b|t_B, s_0)}\ [vp^a_{AC} - Vp^a_{BC}] &\leq Vp^b_{BC} - vp^b_{AC}. \end{aligned}$$

By 1 in the appendix, sincere voting implies $p^a_{AC} = p^b_{BC} > p^a_{BC} = p^b_{AC}$ and $p^a_{AC}/p^a_{BC} \underset{n\to\infty,\ n_C/n=k}{\longrightarrow} \infty$. The above conditions then boil down to:

$$\frac{q(a|t_A, s_0)}{q(b|t_A, s_0)} V \geq v \text{ and } \frac{q(a|t_B, s_0)}{q(b|t_B, s_0)} v \leq V.$$

It is clear that the former condition gets violated for $q(a|t_A, s_0) \to 0$, and the latter gets violated for $q(a|t_B, s_0) \to 1$. It thus remains to show that, for any $\frac{q(a|t,s_0)}{q(b|t,s_0)}$ sufficiently close to zero or to infinity, there is a strategy σ^0 in the neighborhood of $\sigma_{t_A,s_0}(A) = \sigma_{t_B,s_0}(B) = 1$ that ensures that the two conditions (9) and (10) hold strictly. Then, by a continuity argument, we will show that an expectationally stable exists between these two strategies.

Consider the case $q(a)$ very close to 1. This implies that $\frac{q(a|t_B,s_0)}{q(b|t_B,s_0)}$ is very large. Now, consider a strategy $\sigma^0 \equiv \left\{\sigma_{t_A,s_0}(A)=1, \sigma^0_{t_B,s_0}(B)=1-\varepsilon\right\}$, in which t_A voters vote sincerely, and t_B voters strictly mix between A and B. Since $\tau^a_A\left(\sigma^0\right) > \tau^a_B\left(\sigma^0\right)$, condition (9) is still satisfied. What about (10)? It is only satisfied if:

$$\frac{q(a|t_B,s_0)}{q(b|t_B,s_0)}\left[v - V\frac{p^a_{BC}}{p^a_{AC}}\right] - V\frac{p^b_{BC}}{p^a_{AC}} + v\frac{p^b_{AC}}{p^a_{AC}} \leq 0. \tag{11}$$

Holding $n_C/n = k$ constant, we have: $\lim_{n\to\infty}\frac{p^a_{BC}}{p^a_{AC}} = 0$ by 1 in the appendix. It is straightforward to check that the same holds for $\frac{p^b_{AC}}{p^a_{AC}}$ since $\tau^a_A > n_C/n > \tau^b_A$ (remember that we work under the assumption that $r(t_A|a) = r(t_B|b)$). It remains to check how $\frac{p^b_{BC}}{p^a_{AC}}$ varies with n:

$$\frac{p^b_{BC}}{p^a_{AC}} = \left(\frac{\left(\tau^b_B\right)^{k-1/n}\left(1-\tau^b_B\right)^{1-k-1/n}}{\left(\tau^a_A\right)^{k-1/n}\left(1-\tau^a_A\right)^{1-k-1/n}}\right)^n \frac{\frac{\tau^b_B}{k}+\frac{\tau^b_A}{1-k}}{\frac{\tau^a_A}{k}+\frac{\tau^a_B}{1-k}} \underset{n\to\infty}{\to} \left(\frac{\left(\tau^b_B\right)^{k}\left(1-\tau^b_B\right)^{1-k}}{\left(\tau^a_A\right)^{k}\left(1-\tau^a_A\right)^{1-k}}\right)^n \frac{\frac{\tau^b_B}{k}+\frac{\tau^b_A}{1-k}}{\frac{\tau^a_A}{k}+\frac{\tau^a_B}{1-k}}. \tag{12}$$

We now show that the first factor between parentheses must be strictly larger than 1, and hence that $\lim_{n\to\infty}\frac{p^b_{BC}}{p^a_{AC}} = \infty$. First, note that, for the strategy σ^0:

$$\tau^a_A > \tau^b_B > k > 1-k > 1-\tau^b_B > 1-\tau^a_A.$$

Second, note that the function $x^\alpha(1-x)^{1-\alpha}$ is maximized in $x=\alpha$. Thus, the fact that types t_B mix between A and B increases A's vote share in comparison to sincere voting, which (slightly) reduces τ^b_B and brings it closer to k. This increases the numerator. By symmetry, the denominator decreases. Since the first factor is equal to 1 under sincere voting, it must be strictly larger than 1 in σ^0. Thus, there exists $\bar{n}$ large enough such that condition (11) must hold strictly for any n larger than $\bar{n}$.

Having established this, we thus have that:

$$\begin{aligned} G(A|t_A,s_0) - G(B|t_A,s_0) &> 0 \text{ both with sincere voting and at } \sigma^0 \\ G(A|t_B,s_0) - G(B|t_B,s_0) &> 0 \text{ with sincere voting} \\ G(A|t_B,s_0) - G(B|t_B,s_0) &< 0 \text{ at } \sigma^0. \end{aligned}$$

Thus, there must exist σ^* such that $\sigma^*_{t_A,s_0}(A)=1, \sigma^*_{t_B,s_0}(B) \in (1-\varepsilon, 1)$ which is a sincerely stable equilibrium. ■

Proof of Proposition 6. Consider a distribution of types such that $r(t_A|a) > r(t_B|b) > n_C/n$. Holding n_C/n constant, $\exists\bar{n}$ such that the sincere voting equilibrium does not exist when $n > \bar{n}$. Indeed, by (12), we have that, for $\sigma^{\text{sincere}} \equiv \{\sigma_{t_A}(A), \sigma_{t_B}(B)\} = \{1,1\}$, $\lim_{n\to\infty}\frac{p^b_{BC}}{p^\omega_{AC}} = \infty\ \forall\omega$.

From (5), this implies that $G(A|t_A) - G(B|t_A) < 0$.

Now, consider a second strategy profile $\sigma' \equiv \{\sigma'_{t_A}, 1\}$ with $\sigma'_{t_A} \in (0,1)$ such that $\tau_A^a = n_C/n$, which necessarily implies $\tau_A^a < \tau_B^b$. From Lemma 1 and the proof of Proposition 5, this implies that $\lim_{n\to\infty} \frac{p_{BC}^{\omega}}{p_{AC}^{a}} = 0 \; \forall\omega$, and hence that $G(A|t_A) - G(B|t_A) > 0$. This means that the value of $G(A|t_A) - G(B|t_A)$ changes sign when $\sigma_{t_A}(A)$ is increased from σ'_{t_A} to 1. Since all pivot probabilities are continuous in σ_{t_A}, there must exist a value $\sigma^*_{t_A}(A) \in (\sigma'_{t_A}, 1)$ such that voters with signal t_A are indifferent between playing A and B (note that this does not mean that this value is unique). By (7), types-t_B strictly prefers to play B in $\{\sigma^*_{t_A}, 1\}$, which proves that this is an equilibrium.

To prove that the three-candidate equilibrium $\{\sigma_{t_A}(A), \sigma_{s_B}(B)\} = \{\sigma^*_{t_A}(A), 1\}$ is expectationally stable, we just need to show that $\exists\varepsilon_1, \varepsilon_2 > 0$ s.t.

$$\begin{aligned} G(A|t_A) &> G(B|t_A) \text{ if } \sigma_{t_A}(A) \in [\sigma^*_{t_A}(A) - \varepsilon_1, \sigma^*_{t_A}(A)), \\ G(A|t_A) &< G(B|t_A) \text{ if } \sigma_{t_A}(A) \in (\sigma^*_{t_A}(A), \sigma^*_{t_A}(A) + \varepsilon_1], \\ G(A|t_B) &< G(B|t_B) \text{ if } \sigma_{t_B}(B) \in [1 - \varepsilon_2, 1), \end{aligned}$$

From our proof of existence, we know that there is at least one $\sigma^*_{t_A}(A) \in (\sigma'_{t_A}, 1)$ such that (i) $G(A|t_A) > G(B|t_A)$ if $\sigma_{t_A}(A) \in [\sigma^*_{t_A}(A) - \varepsilon)$, and (ii) $G(A|t_A) < G(B|t_A)$ if $\sigma_{t_A}(A) \in (\sigma^*_{t_A}(A) + \varepsilon]$. We also know that $G(A|t_B) < G(B|t_B)$. Therefore, by continuity, it must be that $\exists\varepsilon_2$ such that $G(A|t_B) < G(B|t_B)$ if $\sigma_{t_B}(B) \in [1 - \varepsilon_2, 1)$. This also proves that if the three-candidate equilibrium is unique, then is must be expectationally stable.

Note that the above argument about expectational stability holds for any $q(a) \in (0,1)$. Yet, since it relies on the influence of pivot probabilities in both states of nature, it ceases to hold when $q(a) \in \{0,1\}$. Hence, Aggregate Uncertainty is both a necessary and a sufficient condition for the expectational stability of the equilibrium $\{\sigma^*_{t_A}, 1\}$. ■

Appendix A5: Large Elections, Additional Results

Proposition 7 *Under aggregate certainty, conditional on observing signal $s_\omega \in \{s_a, s_b\}$, if $|r(t_A|\omega) - 1/2| > \delta(n, n_C)$, then there exists a Duverger's Hypothesis equilibrium in which voters with types-t_A play a non-degenerate mixed strategy: $\sigma_{t_A}(A) \in (0,1)$ and $\sigma_{t_B}(B) = 1$. This equilibrium is* ***never*** *expectationally stable.*

Proof. Consider a distribution of types such that $r(t_A|a) - r(t_B|a) > \delta(n, n_C)$, in which case sincere voting is not an equilibrium. That is, there exists a type $\bar{t} \in \{t_A, t_B\}$ such that all the voters of type $\bar{t}$ strictly prefer to deviate from a strategy profile $\sigma^{\text{sincere}} \equiv \{\sigma_{t_A}(A), \sigma_{t_B}(B)\} = \{1,1\}$. First, we prove that types-t_B are the voters who prefer to deviate, i.e. $\sigma^{\text{sincere}} \Rightarrow G(A|t_B) - G(B|t_B) > 0$ for both types. First, note that from $r(t_A|a) - r(t_B|a) > \delta(n, n_C)$, we have that $\tau_A^a > \tau_B^a$ for

σ^{sincere}. Therefore, from $n_C > \frac{n}{2}$, we have

$$\frac{p^a_{AC}}{p^a_{BC}} = \left(\frac{\tau^a_A}{\tau^a_B}\right)^{2n_C - n} \frac{\tau^a_A (n - n_C) + \tau^a_B n_C}{\tau^a_A n_C + \tau^a_B (n - n_C)} > 1.$$

Indeed, for $\tau^a_A = \tau^a_B$, the ratio is equal to 1, and the derivative of this ratio with respect to $\frac{\tau^a_A}{\tau^a_B}$ is proportional to:

$$n_C (n - n_C) \left(1 - \frac{\tau^a_A}{\tau^a_B}\right)^2 + n\frac{\tau^a_A}{\tau^a_B} (n - 1),$$

which is strictly positive. The ratio $\frac{p^a_{AC}}{p^a_{BC}}$ is thus strictly increasing in the share τ^a_A.

Now, consider another strategy profile $\sigma'' \equiv \{\varepsilon, 1\}$, with $\varepsilon \to 0$ (and hence $\sigma_{t_A}(B) \to 1$). From Proposition 1, this strategy profile implies $G(A|t) - G(B|t) < 0$ for both types. By the continuity of the payoffs with respect to $\sigma_{t_A}(A)$, there must therefore exist a value $\sigma^{**}_{t_A}(A) \in (0, 1)$ such that $G(A|t_A) - G(B|t_A) = 0$. It is easy to prove that for the strategy profile $\{\sigma_{t_A}(A), \sigma_{s_B}(B)\} = \left\{\sigma^{**}_{t_A}(A), 1\right\}$, a voter with types-$t_B$ strictly prefers to play B, i.e. $G(A|t_B) - G(B|t_B) < 0$. Hence, that strategy profile is a Duverger's Hypothesis equilibrium.

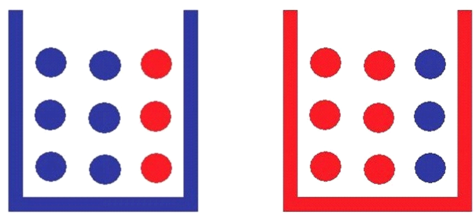
To prove that the equilibrium is not expectationally stable, it is enough to show that for any $\varepsilon > 0$, and $\sigma_t(A) \in [\sigma^*_t(A) - \varepsilon)$, we have $G(A|t_A) < G(B|t_A)$. Note that, for such $\sigma_t(A)$, we have that $\tau^a_A(\sigma_t(A)) < \tau^a_A(\sigma^*_t(A))$. Therefore, the ratio $\frac{p^a_{AC}}{p^a_{BC}}$ is smaller than in equilibrium (see above). This concludes the proof. ■

Appendix A6: Instructions

Welcome and thank you for taking part in this experiment. Please remain quiet and switch off your mobile phone. It is important that you do not talk to other participants during the entire experiment. Please read these instructions very carefully; the better you understand the instructions the more money you will be able to earn. If you have further questions after reading the instructions, please raise your hand out of your cubicle. We will then approach you in order to answer your questions personally. Please do not ask aloud.

This experiment consists of 80 rounds. The rules are the same for all participants and for all rounds. At the beginning of the experiment, you will be randomly assigned to a group of 12 (including yourself). You will belong to the same group throughout the whole experiment. You will only interact with the participants in your group. Your earnings will depend partly on your decisions, partly on the decisions of the other participants in your group and partly on chance.

The Jar. There are two jars: the blue jar and the red jar. The blue jar contains 6 blue balls and 3 red balls. The red jar contains 3 blue balls and 6 red balls.



At the beginning of each round, one of the two jars will be randomly selected. The color of the jar may thus change from one round to the next. In each round, each jar is equally likely to be selected, i.e., each jar is selected with a probability of 50%.

[U] You will not be told which jar has been chosen before making your decision.

[C] You will be told which jar has been chosen before making your decision.

The Ball. After a jar is selected for your group, every participant in your group (including yourself) is going to see the color of one ball randomly drawn from that jar. Since you are 12 in your group, the computer performs this random draw 12 times. Each ball is equally likely to be drawn. That is, independently of the balls received by the other members of the group; you have a chance of two thirds of receiving a blue ball if the selected jar is blue. And if the selected jar is red, you and every other member of your group have a chance of two thirds of receiving a red ball.

Importantly, you will only see the color of your own ball, and not the color of the ball received by the other members of your group.

Your decision. Once you have seen the color of one of the balls, you can make your decision. You will have to vote for Blue, Red or Gray. You can vote for one of the colors by clicking on it. After making your decision, please press the 'OK' key to confirm.

Group Decision. Once all participants have made their decision, the votes of all 12 participants will be added up. On top of that, the computer will add 7 votes for Gray. The group decision will depend on the total number of votes that each color receives:

- If one color has strictly more votes than other colors, this color will be the group decision.

- If there is a tie between several colors with the most votes, one of the colors with the most votes will be selected randomly. Among the colors with the most votes, each color will have the same probability of being chosen to be the group decision.

Payoff in Each Round. Your payoff depends on the group decision and on the color of your ball. Your payoff is indicated in the following table:

		Group Decision		
		Blue	Red	Gray
Your	Blue	200	110	20
Ball	Red	110	200	20

The columns (indicated on the top part of the table) indicate the group decision, i.e. which color got elected. The row (indicated on the left part of the table) indicates the color of your ball.

- If your ball is Blue and the group decision is Blue, you get 200 cents.
- Blue Red, you get 110 cents.
- Blue Gray, you get 20 cents.
- Red Blue, you get 110 cents.
- Red Red, you get 200 cents.
- Red Gray, you get 20 cents.

To summarize, if the color of the group decision matches the color of your ball, your payoff is 200. If group decision is either blue or red, but does not match the color of your selected ball, your payoff is 110. Finally, if the color of the group decision is Gray, your payoff is 20 independently of the color of your ball.

Information at the end of each Round. Once you and all the other participants have made and confirmed your choices, the round will be over. At the end of each round, you will receive the following information:

- Total number of votes for Blue
- Total number of votes for Red
- Total number of votes for Gray (including the 7 votes added by the computer)
- What is the group decision (which color got elected)
- The color of the selected jar
- Your ball
- Your payoff

Final Earnings. At the end of the experiment, the computer will randomly select 8 rounds and you will earn the payoffs you obtained in these rounds. Each of the 80 rounds has the same chance of being selected.

Control Questions. Before the experiment, you will have to answer some control questions in the computer terminal. Click OK after you have answer each question. Once you and all the other participants have answered all the questions, the experiment will start.