# Partial Ambiguity

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**Abstract**

This paper investigates attitude towards partial ambiguity. In a laboratory setting, we study three symmetric variants of the ambiguous urn in Ellsberg's 2-urn paradox by varying the possible compositions of red and black cards in a 100-card deck. Subjects value betting on a deck with a smaller set of ambiguous states more, even when they share the same end points. The valuation of lotteries with two ambiguous states decreases in the degree of spread except for a reversal when it approaches the extreme case of all red or all black. We further study attitude towards skewed ambiguity and find that subjects tend to switch from being ambiguity averse to ambiguity tolerant as the degree of ambiguity goes from moderate to skewed. This paper also discusses the implications of our findings for existing models of decision making under ambiguity including those based on multiple priors, a two-stage perspective, and source preference.

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# 1 Introduction

The classical 2-urn thought experiment of Keynes (1921, p.75) and Ellsberg (1961) suggests that people generally favor betting on an urn with a known composition of 50 red and 50 black balls over betting on another urn with an unknown composition of red or black balls which add to 100. Ellsberg (1961) further suggests a 3-color experiment in which subjects would rather bet on red than on black and bet on not red than not black in an urn with 30 red balls and 60 balls with unknown composition of yellow and black balls. Such preference, dubbed ambiguity aversion, casts doubt on the descriptive validity of subjective expected utility and has given rise to a sizable theoretical and experimental literature (see Camerer and Weber, 1992; Al-Najjar and Weinstein, 2009). Notice that the nature of ambiguity in the three-color paradox with drawing red having a known of chance of $1/3$ versus the chance of drawing yellow (or black) being anywhere between 0 and $2/3$ is skewed relative to that in the 2-urn paradox. While experimental evidence corroborating ambiguity aversion for the 2-urn paradox has been pervasive, the corresponding evidence for the 3-color paradox appears mixed. In their 1985 paper, Curley and Yates examine different comparisons involving skewed ambiguity, e.g., an unambiguous bet of $p$ chance of winning versus an ambiguous bet in which the chance of winning can be anywhere between 0 and $2p$ and observe ambiguity neutrality when the $p$ is less than 0.4. This is corroborated by the finding of ambiguity neutrality in the 3-color urn in two recent papers (Charness, Karni and Levin, 2012; Binmore, Stewart and Voorhoeve, 2012). By contrast, ambiguity affinity for more moderate levels of skewed ambiguity have been observed in Kahn and Sarin (1988) and more recently in Abdellaoui et al. (2011) and Abdellaoui, Klibanoff and Placido (2011).

In their 1964 paper, Becker and Brownson introduce a refinement of the 2-urn paradox to the case of symmetric partial ambiguity with the number of red balls (or black balls) in the unknown urn being constrained to be in a symmetric interval, e.g., $[40, 60]$ or $[25, 75]$ in relation to a fully ambiguous urn of $[0, 100]$ and the $50 - 50$ urn denoted by $\{50\}$. They find that subjects tend to be more averse to bets involving larger intervals of ambiguity.

This motivates us to examine two additional kinds of symmetric ambiguous lotteries. One involves two possible compositions – $\{n\}$ and $\{100-n\}$. Another kind of symmetric partial ambiguity consists of a union of two disjoint intervals $[0,n]\cup[100-n,100]$.

In this paper, we study experimentally attitude towards symmetric partial ambiguity in Part I and attitude towards skewed ambiguity in Part II. The observed patterns of behavior in Part I are summarized as follows:

1. For both interval and disjoint partial ambiguity, we observe aversion to increasing size of ambiguity in terms of the number of possible compositions.

2. The certainty equivalents (CE) of two-point ambiguous lotteries decrease from $\{50\}$ to $\{40,60\}$, from $\{40,60\}$ to $\{30,70\}$, from $\{30,70\}$ to $\{20,80\}$, and from $\{20,80\}$ to $\{10,90\}$ except for the last comparison where the CE increases significantly from $\{10,90\}$ to $\{0,100\}$. Notably, CE of $\{0,100\}$ is not significantly different from that of $\{50\}$.

3. Mean CE of two-point ambiguous lotteries exceeds the mean CE of the interval ambiguous lotteries which in turn exceeds the mean CE of the disjoint ambiguous lotteries.

The design of Part II of our study relates to what is used in Curley and Yates (1985). We find that subjects tend to exhibit a switch in ambiguity attitude from aversion to affinity at around 30% for the known probability. This provides a rationale for the mixed evidence for ambiguity aversion in the 3-color urn. Our finding also echoes a further suggestion of Ellsberg described in footnote 4 of Becker and Brownson (1964). "Consider two urns with 1000 balls each. In Urn 1, each ball is numbered from 1 to 1000, and in Urn 2 there are an unknown number of balls bearing any number. If you draw a specific number say 687, you win a prize. There is an intuition that many subjects would prefer the draw from Urn 2 over Urn 1, that is, ambiguity seeking when probability is small." This intuition has been tested by Einhorn and Hogarth (1985, 1986) in a hypothetical choice study involving 274 MBA students. They find that 19% of their subjects are ambiguity averse with respect to the classical Ellsberg paradox while 35% choose the ambiguous urn when $[0,0.002]$ is the interval of ambiguity rather than the unambiguous urn with an unambiguous winning probability of

0.001.

In the penultimate section of our paper, we shall discuss the implications of our experimental design and the observed choice behavior for various existing models of attitude towards ambiguity. In particular, the comparative behavior of two-point ambiguous and interval ambiguous lotteries which share the same end points has implications on the idea of viewing ambiguity pessimistically in terms of the worst of a set of priors (Wald, 1950; Gilboa and Schmeidler, 1989) as well as its derivatives (Hurwicz, 1951; Ghirardato, Maccheroni and Marinacci, 2004; Maccheroni, Marinacci and Rustichini 2006; Gajdos et al., 2008; Siniscalchi, 2009). Notice that full ambiguity $[0, 100]$ can be viewed as a convex combination of interval ambiguity $[n, 100 - n]$ and disjoint ambiguity $[0, n] \cup [100 - n, 100]$. This property bears on the idea suggested in Becker and Brownson (1964) and Gardenfors (1979) to view ambiguity as the second stage distribution of possible compositions occurring at an initial stage. This idea has been applied by Segal (1987) to account for ambiguity aversion and is subsequently axiomatized in Segal (1990), Klibanoff, Marinacci and Mukerji (2005), Nau (2001, 2006), Seo (2009) and Ergin and Gul (2009). We also study the implications on another view of ambiguity in terms of a limited sense of probabilistic sophistication with red and black regarded as being equally likely (Keynes, 1921; Smith, 1969). This dependence of the decision maker's preference on the underlying source of uncertainty is more formally discussed in Tversky and Kahneman (1992), Fox and Tversky (1995) and Nau (2001). Chew and Sagi (2008) offer an axiomatization of limited probabilistic sophistication over smaller families of events without requiring monotonicity or continuity.

The rest of this paper is organized as follows. Section 2 presents details of our experimental design. Section 3 reports our experimental findings. Section 4 discusses the implications of our experimental findings for a number of decision making models in the literature. Section 5 discusses the related literature and concludes.

## 2 Experimental Design

We use $\{n\}$ to denote an unambiguous deck with a known composition of $n$ red cards and $100-n$ black cards. A fully ambiguous deck is denoted by $[0,100]$. Let $\mathcal{A}$ denote the set of possible compositions in terms of the possible number of red cards in the 100-card deck. Consider the following three symmetric variants of full ambiguity described: interval ambiguity denoted by $[n,100-n]$, two-point ambiguity denoted by $\{n,100-n\}$, and disjoint ambiguity denoted by $[0,n]\cup[100-n,100]$. We further define three benchmark treatments: $B_0=\{50\}$, $B_1=\{0,100\}$, and $B_2=[0,100]$. Here, $B_1$ appears to admit some ambiguity in interpretation. Being either all red or all black may give it a semblance of a $50-50$ lottery in parallel with its intended interpretation as being two-point ambiguous. Interestingly, $B_2$ admits an alternative description as follows. It can first be described as comprising 50 cards which are either all red or all black while the composition of the other 50 cards remains unknown. This process can be applied to the latter 50 cards to arrive at a further division into 25 cards which are either all red or all black while the composition of the remaining 25 cards remains unknown. Doing this ad infinitum gives rise to a dyadic decomposition of $[0,100]$ into subintervals which are individually either all red or all black.

Part I of our study is based on the following 3 groups of six treatments (see Figure 1). In each treatment, subjects choose their own color to bet on.

*Two-point ambiguity.* This involves 6 lotteries with symmetric two-point ambiguity:

$$B_0=\{50\},\ P_1=\{40,60\},\ P_2=\{30,70\},\ P_3=\{20,80\},\ P_4=\{10,90\},\ B_1=\{0,100\}\,.$$

*Interval ambiguity.* This comprises 6 lotteries with symmetric interval ambiguity:

$$B_0=\{50\},\ S_1=[40,60],\ S_2=[30,70],\ S_3=[20,80],\ S_4=[10,90],\ B_2=[0,100].$$

*Disjoint ambiguity.* This involves 6 lotteries with symmetric disjoint ambiguity:

$B_1=\{0,100\}$, $D_1=[0,10]\cup[90,100]$, $D_2=[0,20]\cup[80,100]$, $D_3=[0,30]\cup[70,100]$, $D_4=[0,40]\cup[60,100]$, $B_2=[0,100]$.

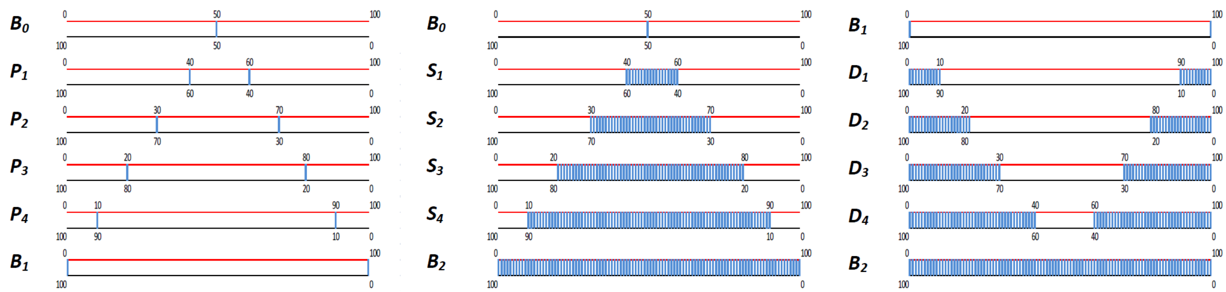

Figure 1. Illustration of 15 treatments in 3 groups.[1]

Part II of our study concerns attitude towards skewed partial ambiguity. It comprises 6 comparisons between two skewed lotteries: $r_n = \{n\}$ and $u_n = [0, 2n]$ where $n = 5, 10, 20, 30, 40$ and 50. Unlike the case of symmetric ambiguity in Part I, subjects here choose between a risk task and an ambiguity task always betting on red.

Both Part I and II lotteries delivers either a winning outcome of S\$40 (about US\$30) or else nothing. To elicit the CE of a lottery in Part I, we use a price list design (e.g., Miller, Meyer, and Lanzetta, 1969; Holt and Laury, 2002), where subjects are asked to choose between betting on the color of the card drawn and getting some certain amount of money. For each lottery, subjects have 10 binary choices corresponding to 10 certain amounts ranging from S\$6 to S\$23. The order of appearance of the 15 lotteries in Part I is randomized for each subject who each makes 150 choices in all. Subsequent to Part I, we conduct Part II of our experiment consisting of 6 binary choices with the order of appearance randomized.

At the end of the experiment, in addition to a S\$5 show-up fee, each subjects is paid based on his/her randomly selected decisions in the experiment. For Part I, one out of 150 choices is randomly chosen using dice. For Part II, one subject is randomly chosen to receive the payment based on one random choice out of his/her 6 binary choices. (see Appendix A for experiment instructions).

[1] Interpretation of the figures is the following: the upper red line represents the number for red cards and the lower black line for black cards, while one vertical blue line represents one possible combination of the deck. Also note that $\{50\}$, $\{0, 100\}$ and $[0, 100]$ are limit cases for different groups.

We are aware that our adoption of a *random incentive mechanism* (RIM) could be subject to violation of the *reduction of compound lottery axiom* (ROCLA) or the independence axiom (e.g., Holt, 1986). In Starmer and Sugden's (1991) study of RIM, they find that their subjects' behavior is inconsistent with ROCLA. More recently, Harrison, Martinez-Correa and Swarthout (2011) test ROCLA specifically and their finding is mixed. While the analysis of choice patterns suggests violations of ROCLA, their econometric estimation suggests otherwise. The use of RIM has become prevalent in part because it offers an efficient way to elicit subjects' preference besides being cognitively simple (see Harrison and Rutstrom 2008 for a review).

We recruited 56 undergraduate students from National University of Singapore (NUS) as participants using advertisement posted in its Integrated Virtual Learning Environment. The experiment consisted of 2 sessions with 20 to 30 subjects for each session. It was conducted by one of the authors with two research assistants. After arriving at the experimental venue, subjects were given the consent form approved by at NUS' institutional review board. Subsequently, general instructions were read to the subjects followed by our demonstration of several example of possible compositions of the deck before subjects began making decisions. After finishing Part I, subjects were given the instructions and decision sheets for Part II. Most subjects completed the decision making tasks in both parts within 40 minutes. At the end of the experiment, subjects received payment based on a randomly selected decision made in addition to a S$5 show-up fee. The payment stage took up about 40 minutes.

# 3 Observed Choice Behavior

This section presents the observed choice behavior at both aggregate and individual levels and a number of statistical findings.

*Part I.* Summary statistics are presented in Figure 2.[2] We apply the Friedman test to check

[2] Out of 15 Part I tasks, one subject exhibits multiple switching in one task and another exhibits multiple switching in three tasks. Their data for these 4 tasks are excluded from our analysis.

whether the CEs of the 15 lotteries come from a single distribution. We reject the null hypothesis that the CEs come from the same distribution ($p < 0.001$). Besides replicating the standard finding – CE of $\{50\}$ is significantly higher than that of $[0, 100]$ (paired Wilcoxon Signed-rank test, $p < 0.001$), our subjects have distinct attitudes towards different types of partial ambiguity. Specifically, for the comparison between $\{50\}$ and $[0, 100]$, 62% of the subjects exhibit ambiguity aversion, 33% of the subjects exhibit ambiguity neutrality, and 5% of the subjects exhibit ambiguity affinity.

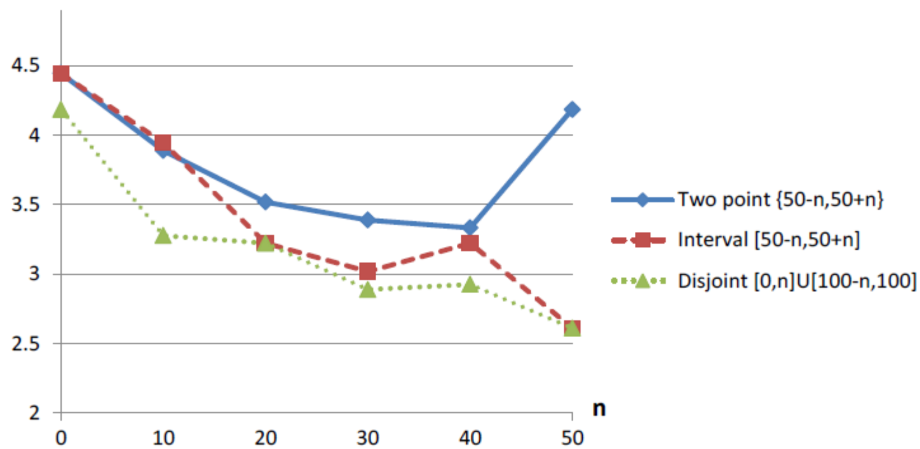


Figure 2. Mean switching points for lotteries in Part I.[3]

The CEs for the 15 lotteries are highly and positively correlated in ranging from 58.8% to 91.6% (see Table S3 in Appendix B for pair-wise Spearman correlations). The correlations between risk attitude measured by the CEs for $B_0 = \{50\}$ and ambiguity attitude, measured by the difference in CEs between that of $B_0$ and those 14 ambiguous lotteries are generally highly correlated, between 36.7% and 63.8%, except for $B_1 = \{0, 100\}$ with a correlation of 9.8% (see Table S4 in Appendix B). The pairwise correlations for the ambiguity attitude towards the 14 ambiguous lotteries are also highly positive, ranging from 55.1% to 87.3%, except for the correlations with $B_1$ which range from 9.6% to 49.2% (see Table S5 in Appendix B). The correlations identified here are similar to those reported in Halevy (2007), and

[3]Data are coded in terms of the number of times each subject chooses the lottery over a sure amount in the 10 binary choices. For details, please refer to Table S1 in Appendix B.

suggest a common link between risk attitude and ambiguity attitude except for $B_1$, which corroborates the earlier observation that it may admit an additional interpretation as being almost a 50-50 lottery.

Using the Trend test, we check subsequently whether there is a significant trend in each group. This yields the following two observations.

Observation 1 (*Interval and disjoint ambiguity*): For lotteries related to interval ambiguity, $B_0$, $S_1$, $S_2$, $S_3$, $S_4$ and $B_2$, there is a statistically significant decreasing trend in CE as size of $\mathcal{A}_S$ increases ($p < 0.001$). For lotteries related to disjoint ambiguity, $B_1$, $D_1$, $D_2$, $D_3$, $D_4$ and $B_2$, there is also a statistically significant decrease in CE as the size of $\mathcal{A}_D$ increases ($p < 0.001$).

Moreover, we count the number of individuals exhibiting specific patterns in Observation 1. For the 6 interval ambiguous lotteries, 24.1% of the subjects have the *same CEs*, 25.9% of the subjects have *non-increasing CEs*, while none of the subjects has *non-decreasing CEs*. For the 6 lotteries in the disjoint ambiguity, 24.1% of the subjects have the same CEs, 20.3% of the subjects have non-increasing CEs, and 5.5% of the subjects have non-decreasing CEs.

Observation 2 (*Two-point ambiguity*): For lotteries related to two-point ambiguity, $B_0$, $P_1$, $P_2$, $P_3$, $P_4$, and $B_1$, there is a significant *nonincreasing trend in the CEs* from $B_0 = \{50\}$ to $P_4 = \{10, 90\}$ ($p < 0.001$). Interestingly, the CE of $B_1$ reverses this trend and is significantly higher than the CE of $P_4$ (paired Wilcoxon Signed-rank test, $p < 0.005$). Moreover, the CE of $B_1$ is not significantly different from that of $B_0$ (paired Wilcoxon Signed-rank test, $p > 0.323$).

At the individual level, for the 6 two-point ambiguity lotteries, 25.9% of the subjects have *the same CEs*, 16.6% of the subjects have *non-increasing CEs*, 22.2% of the subjects have *non-increasing CEs until* $\{10, 90\}$ *with an increase at* $B_1$, and 5.5% of the subjects have *non-decreasing CEs*. Between $B_0$ and $B_1$, 44.5% of the subjects have the same CEs, 35.2% of the subjects display a higher CE for $B_0$ than that for $B_1$, and 23.2% of the subjects exhibit the reverse. Between $B_1$ and $\{10, 90\}$, 44.6% of the subjects have the same CEs, 41.1% of the subjects have a higher CE for $B_1$ than that for $\{10, 90\}$, and 14.3% of the subjects exhibit the reverse, again corroborating the potentially ambiguous nature of $B_1$.

Observation 3 (*Across group*): The mean CE of the lotteries related to two-point ambiguity, $P_1$, $P_2$, $P_3$, $P_4$ and $B_1$, exceeds that of the corresponding lotteries related to interval ambiguity, $S_1$, $S_2$, $S_3$, $S_4$ and $B_2$, given that each pair of $P_i$ and $S_i$ have the same end points ($p < 0.006$). The mean CE of the lotteries related to interval ambiguity, $B_0$, $S_1$, $S_2$, $S_3$ and $S_4$, exceeds that of the corresponding lotteries related to disjoint ambiguity, $B_1$, $D_1$, $D_2$, $D_3$, and $D_4$, given that each pair of $S_i$ and $D_i$ have the same number of possible compositions. ($p < 0.017$).[4]

At the individual level, between two-point ambiguity and interval ambiguity, 25.9% of the subjects have the same mean CEs across the five pairs of lotteries, $\{P_1, S_1\}$, $\{P_2, S_2\}$, $\{P_3, S_3\}$, $\{P_4, S_4\}$ and $\{B_1, B_2\}$, 55.6% of the subjects have higher CE for two-point than for the corresponding interval ambiguity. The rest of 18.5% exhibit the reverse. Between interval ambiguity and disjoint ambiguity, 29.6% of the subjects have the same mean CEs across the five pairs, $\{S_1, D_1\}$, $\{S_2, D_2\}$, $\{S_3, D_3\}$, $\{S_4, D_4\}$ and $\{B_0, B_2\}$, 48.1% of the subjects have higher mean CEs for interval ambiguity than that for the corresponding disjoint ambiguity, and the rest 22.3% of the subjects have the reverse preference.

*Part II.* Figure 3 summarizes the proportion of subjects choosing the ambiguous deck. As anticipated, between $\{50\}$ and $[0, 100]$, a small proportion of 12.5% choose the latter. When the proportion of subjects choosing the ambiguous lottery is significantly lower (higher) than the chance frequency of 0.5, we take the pattern to be ambiguity averse (seeking). Using a simple t-test of difference in proportions, we arrive at the following observation.

Observation 4 (*Skewed ambiguity*): Subjects are significantly averse to moderate ambiguity $[0, 80]$ and $[0, 100]$ ($p < 0.001$ for both cases) and significantly tolerant of skewed ambiguity

[4] Pairwise comparisons of CEs between two-point ambiguity and their corresponding interval ambiguity with the same best and worst priors are not significantly different. In addition, pairwise comparisons between interval ambiguity and their corresponding disjoint ambiguity are also not significant.

for $[0,10]$, $[0,20]$ and $[0,40]$ ($p < 0.002$ in each case). There appears to be a switch towards becoming ambiguity seeking at around $[0,60]$ (marginally significant at $p < 0.105$).

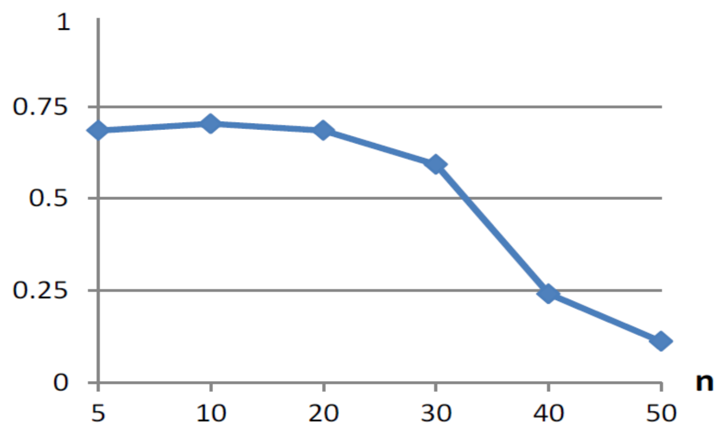


Figure 3. Proportion of subjects choosing the ambiguous lottery $[0,2n]$.[5]

Analyzing the behavior across all 6 choices, 14.3% of the subjects are consistently ambiguity averse, 5.4% are consistently ambiguity seeking, and 39.3% are ambiguity averse towards $[0,80]$ and $[0,100]$ and ambiguity seeking towards $[0,10]$, $[0,20]$ and $[0,40]$.

One issue in the experimental studies of ambiguity is that subjects may feel suspicious that somehow the deck is stacked against them. Such a sentiment may be a confounding factor when eliciting ambiguity attitude. In general, a minimal requirement to control for suspicion would appear to be to let subjects choose which ambiguous event to bet on, e.g., subjects can choose whether to bet on red or black in the 2-color urn. (Einhorn and Hogarth, 1985, 1986; Kahn and Sarin, 1988, Abdellaoui et al., 2011; Abdellaoui, Klibanoff and Placido 2011). For symmetric partial ambiguity in Part I, we control for the effect of suspicion by letting subjects choose which color to bet on. The effect of suspicion is expected to be more pronounced for the lotteries in Part II when subjects only win on drawing a red card. Our data do not appear to offer strong support for this. In Part I, when facing full ambiguity $[0,100]$, 61.1% of the subjects are strictly ambiguity averse, 33.3% are ambiguity neutral, and 5.6% are strictly ambiguity seeking. In Part II, 87.5% choose $\{50\}$ over $[0,100]$ with

[5] For details, please refer to Table S2 in Appendix B.

12.5% making the opposite choice. Moreover, a preponderance of subjects exhibit ambiguity affinity in Part II for three skewed ambiguous lotteries $[0, 5]$, $[0, 10]$, and $[0, 20]$, despite being required to bet on red. Overall, our evidence does not support a clear influence of suspicion in our experiment. This contrasts with the finding of significant influence of suspicion for the case of the 3-color urn in Charness, Karni and Levin (2012) and Binmore, Stewart and Voorhoeve (2012).

Table S6 in Appendix B displays the Spearman correlations in ambiguity attitude of all 6 decisions. We find the correlation between $[0, 100]$ and $[0, 80]$ to be highly positive and that the correlation between $[0, 20]$ and $[0, 10]$ is also highly positive. By contrast, the correlation between $[0, 100]$ and $[0, 10]$ is marginally significantly negative ($p < 0.103$) which is compatible with a good proportion of subjects switching from being ambiguity averse towards the moderate ambiguity of $[0, 80]$ and $[0, 100]$ to being ambiguity seeking for $[0, 10]$, $[0, 20]$, and $[0, 40]$.

# 4 Theoretical Implications

This section discusses the implications of the observed choice behavior for a number of formal models of attitude toward ambiguity in the literature. One approach involves using a nonadditive capacity in place of a subjective probability measure in part to differentiate among complementary events that are revealed to be equally likely (Gilboa, 1987; Schmeidler, 1989). In another approach, attitude towards ambiguity is axiomatized in terms of the decision maker facing a range of priors and being pessimistic or optimistic towards them (Gilboa and Schmeidler, 1989; Ghirardato, Maccheroni and Marinacci, 2004; Maccheroni, Marinacci and Rustichini, 2006; Gajdos, et al., 2008). While related to the multiple priors approach, Siniscalchi's (2009) vector expected utility model is formally distinct. A different axiomatic approach involves evaluating an ambiguous lottery in a two-stage manner (Segal, 1987, 1990; Klibanoff, Marinacci and Mukerji, 2005; Nau, 2006; Seo, 2009; Ergin and Gul, 2009). A related approach is evident in Chew and Sagi's (2008) axiomatization of source preference exhibiting limited probabilistic sophistication in distinguishing between ambiguous states from the unambiguous states.

To facilitate our analysis, we impose the following behavioral assumptions:

*Symmetry (Part I)*: For treatment $i \in \{B_0, ..., P_1, ..., S_1, ..., D_1, ...\}$, the decision maker is indifferent between betting on red and black.

*Conditional Symmetry (Part II)*: For treatment $u_n = [0, 2n]$ with $2n$ cards of unknown color, the decision maker is indifferent between betting on red and black conditional on not having drawn among the $100 - 2n$ black cards.

For the benchmark SEU model or more generally probabilistic sophistication, the probabilities of the events $R_i$ and $B_i$ always equal 0.5 given symmetry where $R_i$ and $B_i$ denote the respective events in treatment $i$. In particular,

$$SEU_i = v(w)/2 + v(0)/2,$$

where $w$ denotes the payment should subjects guess correctly. Thus, SEU predicts that all lotteries in Part I have the same CEs. For Part II, a similar argument based on conditional symmetry implies that $r_n \sim u_n$ for each $n$. Both implications are incompatible with the observed behavior.

## 4.1 Non-additive Capacity Approach

One alternative to SEU, dubbed Choquet expected utility (CEU), is to formulate a non-additive generalization by using a capacity in place of a probability measure (Gilboa, 1987; Schmeidler, 1989). Under CEU, the utility for lottery $i$ is given by:

$$\nu(R_i)v(w) + (1 - \nu(R_i))v(0) = \nu(B_i)v(w) + (1 - \nu(B_i))v(0),$$

with $\nu(R_i) = \nu(B_i)$ from symmetry. In relaxing additivity, the capacities or decision weights assigned to red (or black) for different Part I lotteries need not be the same. At the same time, for unambiguous lotteries, we typically assume that $\nu$ is additive over unambiguous events so that $\nu(R_{\{n\}}) = \widehat{n}$, where $\widehat{n}$ refers to the probability $n/100$. It follows that CEU can generate the pattern of behavior in Part I and Part II if $\nu(\cdot)$ preserves the observed ordering. In particular, for symmetric partial ambiguous lotteries, $\nu(R_i) = \nu(B_i) < 0.5$ for $i \neq B_0$, while $\nu(R_{u_n}) > \widehat{n}$ for $n$ less than 30 and $\nu(R_{u_n}) < \widehat{n}$ for $n$ greater than 30 for skewed ambiguous lotteries.

## 4.2 Multiple Priors Approach

Gilboa and Schmeidler (1989) offer the first axiomatization of the maxmin expected utility (henceforth MEU) specification in which an ambiguity averse decision maker behaves 'as if' there were an opponent who could influence the occurrence of specific states to his/her disadvantage. This intuition is captured by equating the utility of an ambiguous lottery with the expected utility corresponding to the worst prior in a convex set of priors $\Pi$. It is straightforward to see that this model can account for the classical 2-urn Ellsberg paradox. For each treatment $i$, the corresponding set of priors $\Pi_i$ can be viewed as the marginal of $\Pi$ restricted to $\{R_i, B_i\}$. For the Part I lotteries, indifference between betting on red and on black implies that each marginal $\Pi_i$ is symmetric. In the balance of this subsection, we shall be using the subscript $i$ to refer to specific marginals where it applies.

The MEU of lottery $i$ is given by:[6]

$$\min_{\mu\in\Pi_i} \mu\left(R_i\right) v\left(w\right) = \min_{\mu\in\Pi_i} \mu\left(B_i\right) v\left(w\right).$$

It follows that $B_0 \succ P_1 \succ P_2 \succ P_3 \succ P_4 \succ B_1$, $B_0 \succ S_1 \succ S_2 \succ S_3 \succ S_4 \succ B_2$ and $B_1 \sim D_1 \sim D_2 \sim D_3 \sim D_4 \sim B_2$ if we require $\Pi_i$ to depend on the end points of the set of possible compositions. This contradicts our Observations 1, 2 and 3. Without any restriction on the sets of priors, MEU can account for the observed behavior with a judicious choice of the worst prior for each ambiguous lottery.

For Part II, MEU implies that $r_n \succ u_n$ under conditional symmetry, which is incompatible

[6] Note that the utility is the same with $\mu(R)$ or $\mu(B)$ due to symmetry. Thus, we use only $\mu(R)$ for subsequent exposition. We also normalize $v(0) = 0$.

with the observed affinity for sufficiently skewed ambiguity (Observation 4).

Ghirardato, Maccheroni and Marinacci (2004) axiomatize the $\alpha$-MEU model ($\alpha$-M) as a linear combination of maxmin EU and maxmax EU. Their representation, adapted to our setting, is as follows:

$$\alpha_i \min_{\mu \in \Pi_i} \mu(R_i) v(w) + (1 - \alpha_i) \max_{\mu' \in \Pi_i} \mu'(R_i) v(w).$$

Besides inheriting most of the properties of MEU, the implications of $\alpha$-MEU model depend on the value of $\alpha_i$. Suppose $\alpha_i$ is the same for all $i$ and $\Pi_i$ is end-point dependent,[7] MEU and $\alpha$-MEU have the same implications for Part I and Part II if $\alpha > 0.5$. When allowing the freedom of choosing $\alpha$, $\alpha$-MEU can explain all observed patterns. Notably, the observed pattern will impose some monotonicity property of $\alpha$ regarding the size of ambiguity. For example, to explain the aversion to increasing size of ambiguity in the disjoint group $[0, n] \cup [100 - n, 100]$, $\alpha$ needs to be an increasing function of $n$, while for skewed ambiguous lottery $u_n$ in Part II, $a$ needs again to be increasing of $n$ to accommodate the observed switch from ambiguity affinity to aversion. Gajdos, et al. (2008) have axiomatized a closely related model, called a "contraction" model, which delivers a weighted combination between SEU and MEU, and the implications would be similar to those of MEU and $\alpha$-MEU.

Maccheroni, Marinacci and Rustichini (2006) propose an alternative generalization of MEU called Variational Preference (VP) as follows:

$$\min_{\mu \in \Delta} \left\{ \mu(R_i) v(w) + c_i(\mu) \right\},$$

where $c_i(\mu) : \Delta(S) \to [0, \infty)$ is an index of ambiguity aversion. They show that VP could be reduced to MEU if $c_i$ is an indicator function for $\Pi_i$. If we restrict $c_i$ to be the same for all lotteries, then it will imply all lotteries in Part I are the same, which is obviously implausible, while there will be no testable predictions if there are no restrictions on $c_i$. One approach is to make $c_i$ and $c_j$ the same conditioning on the priors that are common to $i$ and $j$, while $c_i$ and $c_j$ each becomes unbounded when the underlying prior does not belong to the respective sets of priors. In this case, we have:

[7] Ghirardato, Maccheroni, and Marinacci (2004) axiomatize this representation where $\alpha$ is constant.

$$\min\left\{VP\left([n,100-n]\right),VP\left([0,n]\cup[100-n,100]\right)\right\}\text{ is constant for all }n,$$

which is rejected by our Observation 1. The other approach permits size-dependent ambiguity index function, if $c_i$ becomes smaller when the size of ambiguity becomes larger, then VP is able to accommodate the observed aversion to increasing size of ambiguity in Part I. But, for Part II, we have $r_n \succ u_n$ for all $n$ under conditional symmetry, which does not fit Observation 4.

Subsequently, Siniscalchi (2009) develops a vector expected utility (VEU) model which relates to VP and can exhibit ambiguity affinity. In our setting, the VEU specification is given by:

$$\mu\left(R_i\right)u\left(w\right)+A\left(\left(\zeta_i\mu\left(R_i\right)u\left(w\right)\right)_{0\leq i<l}\right),$$

where $\zeta_i$ is a real-valued adjustment factor for lottery $i$ and $A:\mathbb{R}^l\rightarrow\mathbb{R}$ is a symmetric function which vanishes at 0. The adjustment term given by $A\left(\left(\zeta_i\mu\left(R_i\right)u\left(w\right)\right)_{0\leq i<l}\right)$ captures attitude towards different sources of ambiguity. VEU reduces to a subclass of VP when $A$ is negative and concave. VEU is compatible with the observed behavior in Part I with $A_i$, the marginal of $A$ restricted to the dimension of lottery $i$, being negative and concave. At the same time, the observed ambiguity affinity in Part II suggests that $A_i$ is positive for values of $\mu\left(R_i\right)$ that are close to 0. This implies that VEU requires more than a countable number of marginal adjustment functions $A_i$ to capture a continual change in attitude towards skewed ambiguity $[0,2n]$ with $n$ varying continuously. While this latter implication is incompatible with the VEU specification, our data based on a finite number of observations cannot directly reveal such a limitation.

## 4.3 Two Stage Approach

The idea of linking ambiguity aversion to aversion to two stage risks coupled with a failure of the reduction of compound lottery axiom (ROCLA) is evident in the works of Becker and Brownson (1964) and Gardenfors (1979). This is formalized in Segal (1987) who proposes a two-stage model of ambiguity aversion with a common rank-dependent utility (Quiggin

1982, henceforth RDU) for both first and second stage risks. Maintaining a two-stage setting without requiring ROCLA, several subsequent papers (Klibanoff et. al., 2005; Nau, 2006; Ergin and Gul, 2009; Seo 2009) provide axiomatizations of a decision maker possessing distinct preferences across the two stages to model ambiguity aversion. We shall discuss successively here the implications of our data on adopting a two-stage approach using both identical and distinct preference specifications for the two stages.

To facilitate our analysis, we impose the following assumption.

*Belief Consistency*: Stage-1 beliefs $\pi_i$ for all $i$ in Part I are updated using Bayesian rule from $\pi_{B_2}$, which has the maximal support in terms of the set of possible compositions.

In fact, symmetry, conditional symmetry together with belief consistency imply uniform stage-1 beliefs $\pi_{\mathcal{A}}$ on the set of possible compositions $\mathcal{A}$ for each ambiguous lottery. We offer an induction based argument as follows. Consider a skewed ambiguous deck $[0,1]$ in which only one card has unknown color. Given conditional symmetry, a decision maker is indifferent between red and black conditioning on this unknown card. This implies that Stage-1 belief $\pi_{[0,1]}$ takes the same value for each possible composition $\{0\}$ and $\{1\}$. Similarly, conditional symmetry and belief consistency implies that $\pi_{[0,2]}(\{0\})$ equals $\pi_{[0,2]}(\{1\})$ which in turn equals $\pi_{[0,2]}(\{2\})$. This argument can be extended to show that stage-1 belief $\pi_{[0,100]}$ assumes the same value for all possible compositions, i.e., stage-1 beliefs are uniform. In the sequel, we discuss the implications of adopting a two-stage approach using both identical and distinct utility with uniform stage-1 beliefs.

Note that with uniform stage-1 belief, it is straightforward that $[0,100]$ could be expressed as a convex combination of $[n,100-n]$ and $[0,n]\cup[100-n,100]$ at stage 1.[8] Thus, a certain class of utility functions, namely those satisfying betweenness axiom (Chew, 1983; Chew, 1989; Gul, 1991) would imply that the CE of $[0,100]$ is between that of $[n,100-n]$ and $[0,n]\cup[100-n,100]$, which is incompatible with Observation 1.

Axiomatizations of two-stage preferences based on non-betweenness preferences have ap-

[8] Assume that the overlapping two points are negligible.

peared in Segal (1990) with the same specification in both stages and Ergin and Gul (2009) whose representation discussed below can accommodate distinct preferences across stages:

$$\Phi\left(\boldsymbol{\pi}_i\right), \text{ where } c_\mu = V^{-1}\left(V\left(w, \mu\left(R_i\right)\right)\right),$$

where $\boldsymbol{\pi}_i$ is the induced distribution of $\pi_i$ on the CE of stage-2 risk $\mu$, and $\Phi, V$ are general utility functions (EU or NEU). Segal's (1987) model corresponds to applying the same RDU specification to both stages of the above expression. He shows that such a decision maker can exhibit ambiguity aversion under certain restrictions on the probability weighting function. Segal's representation is as follows:

$$\int v\left(c_\mu\right) df\left(M_i\right), \text{ where } c_\mu = v^{-1}\left(v\left(w\right) f\left(\mu\left(R_i\right)\right)\right),$$

where $f$ is an increasing probability weighting function, $c_\mu$ is the CE for a stage-2 risk $\mu$, and $M_i$ is the cumulative distribution function of $\pi_i$, which is linear due to uniform belief. Assuming a convex probability weighting function $f$, we have the following implications: $B_0 \succ S_1 \succ S_2 \succ S_3 \succ S_4 \succ B_2$, $B_1 \succ D_1 \succ D_2 \succ D_3 \succ D_4 \succ B_2$ and $B_0 \sim B_1 \succ P_j$.

The intuition for these implications is as follows. For the two-point group $\{n, 100-n\}$, as $n$ deviates from 50, the decision weight on stage-2 risk $\{100-n\}$ becomes $f\left(0.5\right)$, which is less than 0.5 due to the convexity of $f$, thus the evaluation drops at first since the value changes of $\{n\}$ and $\{100-n\}$ relative to $\{50\}$ are the same when $n$ is close to 50. As $n$ decreases, this effect is offset by the effect that the value of $\{100-n\}$ $\left(f\left(1-\widehat{n}\right)v\left(w\right)\right)$ increases faster than the value of $\{n\}$ $\left(f\left(\widehat{n}\right)v\left(w\right)\right)$ drops, which is again due to the convexity of $f$, thus creating a reversal at last. The minimum point occurs at $n^\star$ such that $f'\left(1-\widehat{n}^\star\right)/f'\left(\widehat{n}^\star\right) = \left(1-f\left(0.5\right)\right)/f\left(0.5\right)$, which could be 10 as in Observation 2.

For the interval group $[n, 100-n]$, the intuition is a bit more complicated: as $n$ deviates from 50, the decision weight on the best stage-2 risk $\{100-n\}$ is $f\left(1/\left(2n+1\right)\right)$, which becomes disproportionately smaller compared to that on the other stage-2 risks. To the opposite, the decision weight on the worst stage-2 risk $\{n\}$ is $1-f\left(2n/\left(2n+1\right)\right)$, which becomes disproportionately larger. This effect of changes in decision weights offsets the effect of increasing value of $\{100-n\}$, thus we do not have the reversal when $n$ approaches 0 as in the two-point group. The intuition for the disjoint group is similar.

With the same restrictions on $f$, we can have $r_n \prec u_n$ for $n$ small and $r_n \succ u_n$ for $n$ large.[9] Next, we show by an example that the implications for across-group comparisons under the same restrictions may fail. Consider the lotteries $[49, 51]$ and $\{49, 51\}$, the difference between these two is that the decision weight on $\{50\}$ in lottery $[49, 51]$ is transferred to $\{49\}$ and $\{51\}$ in lottery $\{49, 51\}$, and the transferred weight to $\{51\} : f(1/2) - f(1/3)$, is less than that to $\{49\} : f(2/3) - f(1/2)$, due to the convexity of $f$. Thus, similar intuition as that for the two-point group suggests that $[49, 51] \succ \{49, 51\}$, contradicting Observation 3.

This leads us to apply distinct preferences functionals to the Ergin-Gul specification. We can rule out EU for the second stage as it cannot generate a reversal for the two-point group since the utility for $\{n, 100 - n\}$, given by $f(0.5)(1 - \widehat{n}) v(w) + (1 - f(0.5)) \widehat{n} v(w)$, will then be monotonic in $\widehat{n}$. Building on Segal (1987), we apply distinct RDU's in both stage-1 and stage-2 and consider a convex stage-1 probability weighting function $f$ and a piecewise linear stage-2 probability weighting function $g$ connecting 0 to $f(0.5)$ and $f(0.5)$ to 1. As with Segal (1987), this model can account for Observation 1. For across-group comparison, the utility for a two-point ambiguous lottery $\{n, 100 - n\}$ becomes $f(0.5) g(1 - \widehat{n}) v(w) + (1 - f(0.5)) g(\widehat{n}) v(w)$ which is constant, and will be higher than the utility for the interval group $[n, 100 - n]$, which is monotonically decreasing. We may further perturb the function $g$ to be strictly convex and obtain a reversal in the two-point group such that this model can accommodate all observed patterns.

Several recent papers axiomatize a two-stage model involving distinct EU's in both stages (DEU) including Klibanoff, Marinacci and Mukerji (2005), Nau (2006), and Seo (2009). As observed earlier, DEU is incompatible with Observation 1 since independence implies betweenness.[10] Otherwise, DEU can exhibit ambiguity aversion or ambiguity affinity depending on the relative concavity between the stage-1 and stage-2 vNM utility functions but not

[9] See Segal (1987) problem 3 for an example

[10] The assumption of second order independence in Klibanoff, Marinacci and Mukerji (2005) has been recently discussed in Epstein (2010) and Klibanoff, Marinacci and Mukerji (2012).

their concurrence. Given that stage-1 utility is more concave, DEU can account for observed aversion in the two-point group except for the reversal at $B_1$, but not Observation 3 since it implies that each lottery in the interval group is preferred to the corresponding lottery in the two-point group.

### 4.4 Source Preference Approach

Chew and Sagi's (2008) axiomatize a source preference model which delivers endogenously a one-stage representation for the benchmark lotteries, $B_0$, $B_1$, and $B_2$, and a two-stage representation for the various forms of partial ambiguity in which the unknown red and black cards form a conditional small world while the rest forms the other one.[11] Notice that source view can accommodate the two-stage approach by assuming that all states $\{n\}$ are equally likely. In the following, we derive the implications of our data for a different application of source preference approach to the various partial ambiguity treatments.

*Interval Ambiguity* ($[50-n, 50+n]$): The two end-intervals whose total length is $100-2n$ are known – half red and half black – while the interval portion with length $2n$ is ambiguous. Assume that events of equal length within "known" and within "ambiguous" are equally likely, the overall CE is given by:

$$v^{-1}\left(2\widehat{n}v\left(d\right)+\left(1-2\widehat{n}\right)v\left(c\right)\right),$$

where $v$ is the stage-1 utility and $c = CE_k\left(\frac{1}{2}\delta_w + \frac{1}{2}\delta_0\right)$ and $d = CE_u\left(\frac{1}{2}\delta_w + \frac{1}{2}\delta_0\right)$ with $CE_k$ and $CE_u$ as the CE functionals for known and ambiguous domains respectively. Here, $c > d$ corresponds to ambiguity aversion.

*Disjoint Ambiguity* ($[0, n] \cup [100-n, 100]$): Either of the two end intervals with length $n$ is ambiguous, while the remainder with length $100-n$ is either all red or all black. Assume that events of equal length within "either or" are equally likely, the CE is given by:

$$v^{-1}\left(\widehat{n}v\left(d\right)+\left(1-\widehat{n}\right)v\left(c'\right)\right),$$

[11] A conditional small world refers to a smaller family of events within which the decision maker exhibits probabilistic sophitication.

where $c' = CE_e\left(\frac{1}{2}\delta_w + \frac{1}{2}\delta_0\right)$ with $CE_e$ as the CE functional for the either all red or all black domain. Here, $c' > d$ corresponds to another form of ambiguity aversion which matches the observed pattern of $B_1 \succ B_2$. Notice that the above expression for CE converges to $v^{-1}(0.5v(d) + 0.5v(c'))$ rather than $d$ as $n$ approaches 50. This appears related to the discussion in Section 2 on the dyadic decomposition of $[0, 100]$ into subintervals which are individually either all red or all black. For the source model to deliver the same CE for $B_2$, we need to restrict its evaluation to undecomposed intervals of ambiguity.

The above implication of discontinuous behavior at $n = 50$ does not appear to be compatible with the relatively smooth change of CE for the disjoint group in the overall data. This suggests that subjects may view the size of ambiguity in $[0, n] \cup [100 - n, 100]$ as being $2n$, while viewing $100 - 2n$ as being either all red or all black. This behavior may arise from a decision maker who may possess different valuations for different decompositions of the full ambiguous lottery. Should subjects possess this incorrect understanding, the CE would be given by:

$$v^{-1}\left(2\widehat{n}v(d) + (1 - 2\widehat{n})\, v(c')\right),$$

which will converge continuously to $d$ for the full ambiguity case.

*Two-point Ambiguity* ($\{50 - n, 50 + n\}$): The two end intervals whose total length is $100 - 2n$ are known - half red and half black - while the interval portion $2n$ is either all red or all black, and the CE is given by:

$$v^{-1}\left(2\widehat{n}v(c') + (1 - 2\widehat{n})\, v(c)\right).$$

We have the following implications on the source model.

1. For interval, disjoint and two-point ambiguity, monotonicity alone of $v$ implies $B_0 \succ S_1 \succ S_2 \succ S_3 \succ S_4 \succ B_2$ , $B_1 \succ D_1 \succ D_2 \succ D_3 \succ D_4 \succ B_2$ and $B_0 \succ P_1 \succ P_2 \succ P_3 \succ P_4 \succ B_1$.
2. $P_j \succ S_j \succ D_j$ if $c \geq c' \geq d$ under the misperceived view of $D_j$.

Note that these two implications holds with only probabilistic sophistication on stage-1 utility, which can take a variety of forms besides EU.

3. For Part II, the CE is $v^{-1}(2\widehat{n}v(d))$ for $u_n$, and $c_n = CE_k(\widehat{n}\delta_w + (1 - \widehat{n})\,\delta_0)$ for its

risk counterpart $r_n$. When $n$ is small, suppose we have $c_n > \widehat{n}w > 2\widehat{n}d$ (the first inequality corresponds to risk seeking while the second inequality corresponds to ambiguity aversion). Then $v(c_n) > v(2\widehat{n}d) > 2\widehat{n}v(d)$ if $v$ is concave, which is incompatible with Observation 4. With stage-1 utility taking on a NEU form, the specification can exhibit the desired behavior. For example, with stage-1 weighted utility (Chew 1983), CE for $u_n$ becomes $v^{-1}(2\widehat{n}\omega(d)v(d)/(2\widehat{n}\omega(d)+1-2\widehat{n}))$, which can accommodate the observed affinity in small probability and aversion in moderate probabilities if the weighing function satisfies $\omega(d) > 1$, after normalizing $\omega(0) = \omega(w) = 1$. Alternatively, with stage-1 RDU, CE for $u_n$ is then given by $v^{-1}(f(2\widehat{n})v(d))$, which can also fit Observation 4 if probability weighting function $f$ exhibits an inverted $S$ shape.

### 4.5 Summary

Without further assumptions to render more tractability, most models discussed above can explain a good range of the observed behavior. The implications of our data on the various models, based on additional assumptions that are specific to them, are summarized in Table 1 below. Before discussing the specific implications, it is instructive to revisit the behavior of the two-point ambiguous lotteries $\{n, 100-n\}$ as they approach $B_1 = \{0, 100\}$ whose all red or all black nature may give it some semblance of $B_0 = \{50\}$. Among subjects with nonconstant CE's for two-point ambiguous lotteries, 22.4% assign nonincreasing CE's as $n$ approaches 0 while 30.0% assign nonincreasing CE's only until $n$ equals 10 when there is a reversal with the CE being close to that of $B_0$. The data suggest that some subjects see $B_1$ as $B_0$ which may conceivably be due to its all red or all black nature. This observed reversal in valuation of the two-point group runs counter to several models of ambiguity reviewed in this section. One way to address this reversal is to posit that some subjects do view $B_1$ and $B_0$ as being similar and assign them similar values for their CE's. This accounts for the ticks with asterisks in Table 1 below for several models which otherwise predict monotonic behavior for the CE's in the two-point group.

As anticipated, EU fails to explain all of the observed behavior. On the other hand, without restrictions on its associated capacity, CEU has the flexibility to accommodate all the observed choice behavior. Both MEU and VP exhibit ambiguity aversion globally and cannot account for the observed affinity for skewed ambiguity. Assuming that the set of priors is fully determined by the end points of the various forms of symmetric partial ambiguity, MEU cannot account for the changes in valuations of the disjoint and the two-point ambiguous lotteries as describe in Observation 1 and 2. This observation also applies to the contraction model (not displayed separately in the summary table) which behaves essentially the same as MEU in our experimental setting. In allowing the $\alpha$ parameter to depend on the underlying act, $\alpha$-MEU can account for the switch from ambiguity aversion to ambiguity affinity in Part II. Like MEU and VP, assuming end-point dependence of the set of priors and fixing the value of $\alpha$, $\alpha$-MEU cannot account for Observation 1. While VEU can capture the observed attitude towards symmetric partial ambiguity as well as skewed ambiguity separately, it cannot in principle account for their concurrence in which ambiguity affinity can occur for each point $\{p\}$ over some interval $(0, q)$ for some $q < 1/2$.

The two-stage approach can account for the patterns of observed choice behavior when the two within-stage preferences are represented by distinct non-betweenness specifications such as RDU. When both stages have the same RDU specification, the resulting two-stage model can account for much of the observed behavior except for across group comparisons. When both stages have EU preference, the two-stage approach has a nondescriptive prediction that full ambiguity is intermediate in preference between symmetric and disjoint ambiguity for the same $n$. The case of identical EU preference across the two stages coincides with classical EU under ROCLA, and is similarly unable to account for the observed choice behavior. In treating ambiguous and nonambiguous small domains distinctly, the source preference approach can account for the observed choice behavior except for the observed reversal in the certainty equivalent of the two-point group as it approaches the benchmark lottery of all or nothing.

In Table 1 below, a cross with an asterisk indicates a failure to account for the corresponding observed behavior under some specific auxiliary condition discussed in the preceding subsections.

| Attitude towards Partial Ambiguity | | | CEU | VEU | Multiple Priors | | | Two-Stage | | | | Source |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | Same | | Distinct | | |
| | | | | | VP | αM | MEU | EU | NEU | EU | NEU | |
| Part I | *Obs* 1 | Aversion to size of $A_S$ and $A_D$ | ✓ | ✓ | x* | x* | x* | x | ✓ | x* | ✓ | ✓ |
| | *Obs* 2 | Aversion to spread of $A_P$ except at $B_1$ | ✓ | ✓ | ✓ | ✓* | ✓* | x | ✓ | ✓* | ✓ | ✓* |
| | *Obs* 3 | 2-Point > Interval > Disjoint | ✓ | ✓ | ✓ | x* | x* | x | x* | x* | ✓ | ✓ |
| Part II | Concurrence of ambiguity aversion and affinity | | ✓ | x* | x | ✓* | x | x | ✓ | x | ✓ | ✓ |

**under additional auxiliary conditions*

Table 1. Summary of implications of our data for different models

# 5 Conclusion

Much of the research following Ellsberg (1961) has tended to focus on ambiguity aversion in an all or nothing fashion – either fully known or fully ambiguous (see review in the introduction) with few exceptions, e.g., Becker and Brownson (1964) and Curley and Yates (1985). In this paper, we introduce novel variants of partial ambiguity, namely two-point ambiguity and disjoint ambiguity, study attitude towards partial ambiguity experimentally, and discuss the implications of the observed behavior on a number of models of ambiguity attitude. Our results contribute to a growing experimental literature on testing various models of decision making under uncertainty. Hayashi and Wada (2011) make use of a 'snakes and ladder' game and find evidence against the descriptive validity of MEU. Using a design involving the two-color urn being drawn twice with replacement, Yang and Yao (2011) show that both MEU and DEU inherit specific implications which are incompatible with observed behavior. Machina (2009) offers several examples of Ellsbergian variants which are tested experimentally in L'Haridon and Placido (2010). Their findings are shown in Baillon, L'Haridon, and Placido (2011) to violate the implications of MEU, DEU, VP, CEU, but not VEU. Machina (2009) points out that source preference can account for his examples. Dillenberger and Segal (2012) show that a two-stage NEU representation can also account for Machina's (2009) examples.[12]

Partial ambiguity offers a potentially fruitful avenue to extend existing models to situations where the information possibilities are incomplete or conflicting. Consider an example due to Gardenfors and Sahlin (1982):

> *Consider Miss Julie who is invited to bet on the outcome of three different tennis matches. As regards match A, she is very well-informed about the two players. Miss Julie predicts that it will be a very even match and a mere chance will determine the winner. In match B, she knows nothing whatsoever about the relative strength of the contestants, and has no other information that is relevant for predicting the winner of the match. Match C is similar to match B except that Miss Julie has happened to hear that one of the contestants is an excellent tennis player, although she does not know anything about which player it is, and that the second player is indeed an amateur so that everybody considers the outcome of the match a foregone conclusion.*

The kind of risks illustrated in this example – match A for known risk, match B for interval ambiguity, and match C for disjoint ambiguity – seem representative of what we observe in addition to the entrepreneurial risks as suggested by Knight (1921). Moreover, attitude towards skewed ambiguity, especially the extreme ones, is of particular interest when one concerns designing lottery tickets (Quiggin, 1991) such as whether consumers with skewed ambiguity affinity may prefer pari-mutuel bets over fixed odd bets. Finally, we note that the notion of partial ambiguity can be used in domains where ambiguity aversion has been successfully applied, including finance (Epstein and Wang, 1994; Epstein and Schneider, 2008; Mukerji and Tallon, 2001), contract theory (Mukerji, 1998), and game theory (Lo, 1996; Marinacci, 2000).

[12] Some experimental studies of behavior relating to ambiguity attitude include those linking it to compound lotteries (Yates and Zukowski, 1976; Chow and Sarin, 2002; Halevy, 2007; Abdellaoui, Klibanoff and Placido, 2011; Miao and Zhong, 2012) and those linking it to source preference and familiarity bias (Tversky and Kahneman 1992; Chew et al., 2008; Abdelloui et al., 2011; Chew, Ebstein and Zhong, 2012).

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# A Instructions

DECISION MAKING STUDY

GENERAL INSTRUCTIONS

Welcome to our study on decision making. The descriptions of the study contained in this instrument will be implemented **fully and faithfully**.

**Each participant will receive on average $20 for the study. The overall compensation includes a $5 show up fee in addition to earnings based on how you make decisions.**

**All information provided will be kept CONFIDENTIAL.** Information in the study will be used for research purposes only. Please refrain from discussing any aspect of the specific tasks of the study with any one.

1. The set of decision making tasks and the instructions for each task are the same for all participants
2. It is important to read the instructions CAREFULLY so that you understand the tasks in making your decisions.
3. At ANY TIME, if you have questions, please raise your hand.
4. PLEASE DO NOT communicate with others during the experiment.
5. Do take the time to go through the instructions carefully in making your decisions.
6. Cell phones and other electronic communication devices are not allowed.

INSTRUCTION FOR PART I

This is the first of two parts for today's study. It is made up of 15 decision sheets. Each decision sheet is of the form illustrated below.

| | Option A | Option B | Decision |
|---|---|---|---|
| 1 | A | B1 | A B |
| 2 | A | B2 | A B |
| 3 | A | B3 | A B |
| 4 | A | B4 | A B |
| 5 | A | B5 | A B |
| 6 | A | B6 | A B |
| 7 | A | B7 | A B |
| 8 | A | B8 | A B |
| 9 | A | B9 | A B |
| 10 | A | B10 | A B |

Each such table lists 10 choices to be made between a fixed Option A and 10 different Option B's.

Option A involves a lottery, guessing the color of a card randomly drawn from a deck of 100 cards with different compositions of red and black. If you guess correctly, you receive $40; otherwise you receive nothing. Different tasks will have different compositions of red and black cards as described for each task.

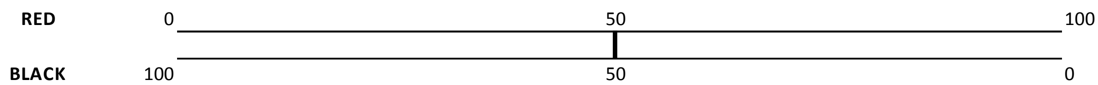
The Option B's refer to receiving the specific amounts of money for sure, and are arranged in an ascending manner in the amount of money.

For each row, you are asked to indicate your choice in the final "Decision" column – A or B – with a tick ($\surd$).

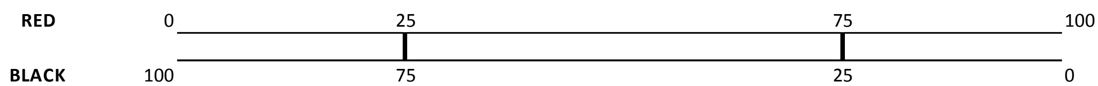
**Examples of Option A**

Each example involves your drawing a card randomly from a deck of 100 cards containing red and black cards.

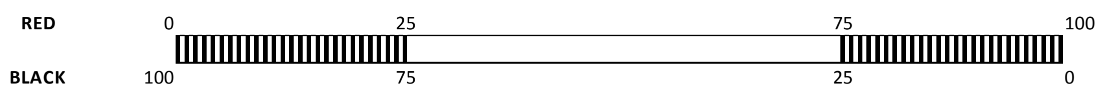
**Example 1**: The deck has 50 red cards and 50 black cards as illustrated below.



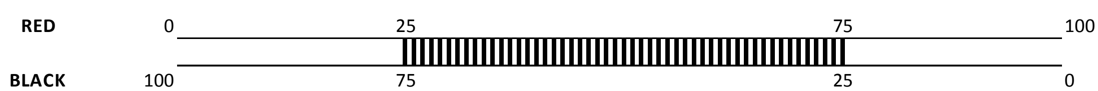

**Example 2**: The deck has either 25 or 75 red cards with the rest of the cards black, as illustrated below.




**Example 3**: The number of red cards may be anywhere between 0 and 25 or between 75 and 100 with the rest of the cards black, as illustrated below.

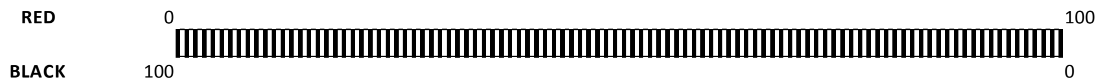



**Example 4**: The number of red cards may be anywhere between 25 and 75 with the rest of the cards black, as illustrated below.




**Example 5**: The number of red cards is anywhere between 0 and 100 with the rest of the

cards black, as illustrated below.

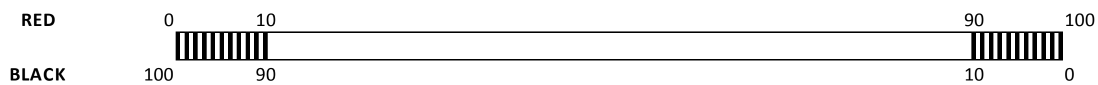



**Selection of decision sheet to be implemented**: One out of the 15 Decision Sheets (selected randomly by you) will be implemented. Should the sheet be chosen, one of your 10 choices will be further selected randomly and implemented.

*Sample Decision Sheet of PART I*

PART I DECISION SHEET - DECK [0-10]U[90-100]

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The number of red cards may be anywhere from 0 to 10 or from 90 to 100 with the rest of the cards black, as illustrated below.




Option A: You guess the color first. You then draw a card from the deck of cards constructed in the above described manner. If you guess the color correctly, you receive \$40. Otherwise, you receive \$0.

The Option B column lists 10 amounts each corresponding to what you will receive for sure if you choose this option.

DECISION: For each of the 10 rows, please indicate your decision in the final column with a tick (√).

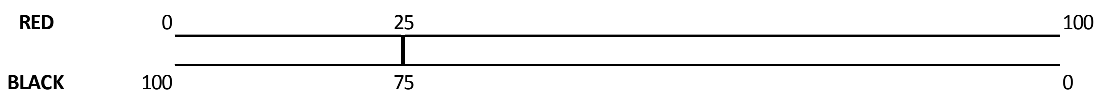
| | Option A | Option B | Decision |
|---|---|---|---|
| 1 | Betting on the cards | Receiving \$6 for sure | A B |
| 2 | Betting on the cards | Receiving \$9 for sure | A B |
| 3 | Betting on the cards | Receiving \$11 for sure | A B |
| 4 | Betting on the cards | Receiving \$13 for sure | A B |
| 5 | Betting on the cards | Receiving \$14 for sure | A B |
| 6 | Betting on the cards | Receiving \$15 for sure | A B |
| 7 | Betting on the cards | Receiving \$16 for sure | A B |
| 8 | Betting on the cards | Receiving \$18 for sure | A B |
| 9 | Betting on the cards | Receiving \$20for sure | A B |
| 10 | Betting on the cards | Receiving \$23 for sure | A B |

INSTRUCTION FOR PART II

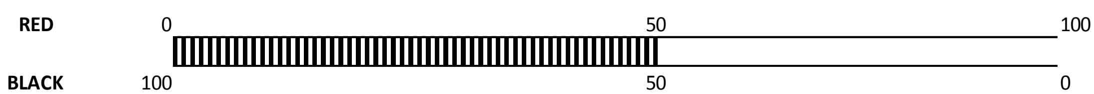
This is the second and final part for today's study. In this part, you will make 6 binary choices. At the end of this part, one of you will be randomly chosen to receive the outcome of one of his/her decisions, also randomly chosen, out of the 6 binary choices made.

Example: Consider Option A and Option B below.

Option A: This bet involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The deck has 25 red cards, and 75 black cards. If you draw the red card, you win $40, otherwise you win nothing.




Option B: This bet involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The number of red cards may be anywhere from 0 to 50 with the rest of the cards black. If you draw the red card, you win $40, otherwise you win nothing.



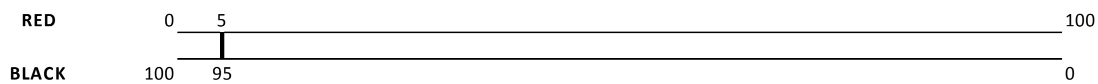

Please circle your choice: A B

*Sample Decision Sheet of PART II*

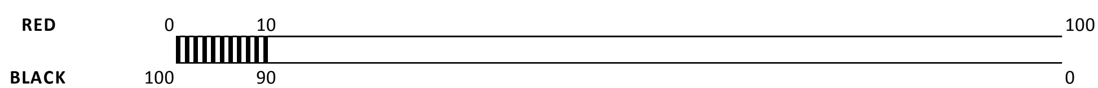
DECISION SHEET FOR PART II

Consider Option A and Option B below.

Option A: This bet involves your drawing a card randomly from a deck of 100 cards containing 5 red cards and 95 black cards. If you draw a red card, you receive $40. Otherwise, you receive $0.




Option B: This bet involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The number of red cards may be anywhere from 0 to 10 with the rest of the cards black. If you draw a red card, you receive $40. Otherwise, you receive $0.




Please circle your choice: A B

# B Supplementary Tables

| Two-point | | | | Interval | | | | Disjoint | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Lottery* | *Mean* | *S.E* | *N* | *Lottery* | *Mean* | *S.E* | *N* | *Lottery* | *Mean* | *S.E* | *N* |
| $B_0$ | **4.51** | 0.42 | 55 | $B_0$ | **4.51** | 0.42 | 55 | $B_1$ | **4.16** | 0.46 | 56 |
| $P_1$ | **3.96** | 0.40 | 56 | $S_1$ | **3.96** | 0.38 | 56 | $D_1$ | **3.31** | 0.37 | 56 |
| $P_2$ | **3.55** | 0.37 | 56 | $S_2$ | **3.34** | 0.35 | 56 | $D_2$ | **3.29** | 0.35 | 56 |
| $P_3$ | **3.40** | 0.37 | 55 | $S_3$ | **3.04** | 0.34 | 55 | $D_3$ | **2.98** | 0.32 | 55 |
| $P_4$ | **3.39** | 0.38 | 56 | $S_4$ | **3.36** | 0.40 | 56 | $D_4$ | **3.04** | 0.35 | 56 |
| $B_1$ | **4.16** | 0.46 | 56 | $B_2$ | **2.71** | 0.37 | 56 | $B_2$ | **2.71** | 0.37 | 56 |

Table S1. Summary statistics of switching points for Part I lotteries.

| *Lottery* | *Mean* | *S.E.* | *N* |
|---|---|---|---|
| [0,100] | 0.125 | 0.044 | 56 |
| [0, 80] | 0.250 | 0.057 | 56 |
| [0, 60] | 0.607 | 0.065 | 56 |
| [0, 40] | 0.696 | 0.061 | 56 |
| [0, 20] | 0.696 | 0.061 | 56 |
| [0, 10] | 0.696 | 0.061 | 56 |

Table S2. Proportion of subjects choosing the ambiguous lottery.

| | B0 | P1 | P2 | P3 | P4 | B1 | S1 | S2 | S3 | S4 | B2 | D1 | D2 | D3 | D4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| B0 | 1 | | | | | | | | | | | | | | |
| P1 | 0.835 | 1 | | | | | | | | | | | | | |
| P2 | 0.851 | 0.913 | 1 | | | | | | | | | | | | |
| P3 | 0.675 | 0.783 | 0.838 | 1 | | | | | | | | | | | |
| P4 | 0.668 | 0.756 | 0.744 | 0.886 | 1 | | | | | | | | | | |
| B1 | 0.802 | 0.780 | 0.760 | 0.679 | 0.777 | 1 | | | | | | | | | |
| S1 | 0.850 | 0.878 | 0.881 | 0.796 | 0.785 | 0.796 | 1 | | | | | | | | |
| S2 | 0.789 | 0.882 | 0.876 | 0.752 | 0.696 | 0.665 | 0.860 | 1 | | | | | | | |
| S3 | 0.719 | 0.852 | 0.862 | 0.740 | 0.648 | 0.588 | 0.836 | 0.912 | 1 | | | | | | |
| S4 | 0.624 | 0.769 | 0.782 | 0.857 | 0.791 | 0.608 | 0.783 | 0.848 | 0.780 | 1 | | | | | |
| B2 | 0.634 | 0.772 | 0.781 | 0.712 | 0.682 | 0.560 | 0.721 | 0.795 | 0.888 | 0.768 | 1 | | | | |
| D1 | 0.761 | 0.780 | 0.839 | 0.828 | 0.805 | 0.709 | 0.822 | 0.777 | 0.706 | 0.816 | 0.712 | 1 | | | |
| D2 | 0.725 | 0.784 | 0.769 | 0.793 | 0.785 | 0.659 | 0.817 | 0.871 | 0.787 | 0.878 | 0.759 | 0.821 | 1 | | |
| D3 | 0.761 | 0.791 | 0.822 | 0.758 | 0.757 | 0.667 | 0.798 | 0.842 | 0.793 | 0.829 | 0.777 | 0.788 | 0.912 | 1 | |
| D4 | 0.692 | 0.781 | 0.785 | 0.749 | 0.685 | 0.612 | 0.786 | 0.844 | 0.806 | 0.830 | 0.832 | 0.807 | 0.916 | 0.876 | 1 |

Table S3. Spearman correlation of CEs for lotteries in Part I.

| Ambiguity | P1 | P2 | P3 | P4 | **B1** | S1 | S2 | S3 | S4 | B2 | D1 | D2 | D3 | D4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Risk | 0.364 | 0.439 | 0.484 | 0.515 | **0.098** | 0.411 | 0.608 | 0.634 | 0.471 | 0.567 | 0.619 | 0.638 | 0.580 | 0.504 |

Table S4. Spearman correlation of risk attitude and ambiguity attitude in Part I.

| | P1 | P2 | P3 | P4 | **B1** | S1 | S2 | S3 | S4 | B2 | D1 | D2 | D3 | D4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| P1 | 1 | | | | | | | | | | | | | |
| P2 | 0.752 | 1 | | | | | | | | | | | | |
| P3 | 0.725 | 0.873 | 1 | | | | | | | | | | | |
| P4 | 0.709 | 0.725 | 0.794 | 1 | | | | | | | | | | |
| B1 | **0.301** | **0.219** | **0.285** | **0.492** | **1** | | | | | | | | | |
| S1 | 0.733 | 0.712 | 0.642 | 0.696 | **0.313** | 1 | | | | | | | | |
| S2 | 0.652 | 0.667 | 0.656 | 0.644 | **0.277** | 0.697 | 1 | | | | | | | |
| S3 | 0.681 | 0.722 | 0.661 | 0.621 | **0.096** | 0.719 | 0.830 | 1 | | | | | | |
| S4 | 0.720 | 0.737 | 0.693 | 0.617 | **0.288** | 0.685 | 0.830 | 0.782 | 1 | | | | | |
| B2 | 0.650 | 0.755 | 0.721 | 0.716 | **0.126** | 0.640 | 0.754 | 0.866 | 0.736 | 1 | | | | |
| D1 | 0.551 | 0.620 | 0.624 | 0.610 | **0.111** | 0.647 | 0.764 | 0.851 | 0.720 | 0.871 | 1 | | | |
| D2 | 0.655 | 0.761 | 0.699 | 0.714 | **0.168** | 0.676 | 0.836 | 0.865 | 0.801 | 0.856 | 0.790 | 1 | | |
| D3 | 0.653 | 0.647 | 0.655 | 0.703 | **0.241** | 0.718 | 0.857 | 0.840 | 0.832 | 0.814 | 0.832 | 0.870 | 1 | |
| D4 | 0.556 | 0.805 | 0.784 | 0.686 | **0.289** | 0.600 | 0.607 | 0.636 | 0.674 | 0.714 | 0.663 | 0.591 | 0.615 | 1 |

Table S5. Spearman correlation of ambiguity attitudes in Part I.

| | [0,100] | [0,80] | [0,60] | [0,40] | [0,20] | [0,10] |
|---|---|---|---|---|---|---|
| [0,100] | 1 | | | | | |
| [0,80] | 0.530 | 1 | | | | |
| [0,60] | 0.194 | 0.380 | 1 | | | |
| [0,40] | 0.132 | 0.112 | 0.423 | 1 | | |
| [0,20] | 0.015 | 0.112 | 0.344 | 0.578 | 1 | |
| [0,10] | **-0.220** | **-0.067** | **0.344** | **0.324** | **0.578** | 1 |

Table S6. Spearman correlation of ambiguity attitudes in Part II.