Multidimensional Ellsberg*

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Abstract

The classical Ellsberg experiment presents individuals with a choice problem in which the probability of winning a prize is unknown (ambiguous). In this paper we study how individuals make choices between gambles in which the ambiguity is in different dimensions: the winning probability, the amount of the prize and the payment date, and many combinations. While the decision-theoretic models accommodate a rich variety of behaviors, we present experimental evidence that points at one systematic behavioral pattern: (i) no ambiguity is preferred to ambiguity on any single dimension and to ambiguity on multiple dimensions, and (*ii*) "correlated" ambiguity on multiple dimensions is preferred to ambiguity on any single dimension.

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1 Introduction

When making decisions under uncertainty we are often confronted with incomplete objective information, or ambiguity, on multiple aspects of the decision problem. The amount of monetary earnings may be ambiguous; the likelihood of the possible earnings may be ambiguous; the date at which payments will be made may be ambiguous; *etc.*. Our willingness to take on ambiguity may depend not only on which dimensions are ambiguous, but also on the extent to which our decision can affect the level of ambiguity in the different dimensions. The goal of this paper is to gain some understanding of how individuals approach this form of "multi-dimensional" ambiguity.

The starting point of modern analysis of decision making under ambiguity is the seminal thought experiment of Ellsberg. In one of its variants, participants faced an urn with 60 poker chips: 20 of these chips are black, while each of the remaining 40 chips is either red or green (red and green are referred to as the "ambiguous" colors). One of these chips would be randomly drawn and participants were asked to guess the color of the chip. If his guess is correct, he would win a prize of \$20, which would be paid immediately.

There are two features of this experiment that we should emphasize. First, all the gambles involve either no ambiguity, or ambiguity in only one dimension: the likelihood of winning a known amount of money. Other dimensions, such as the amount won (if won), or the date of payment, are known. Second, by choosing to bet on Black, the participant can select a gamble with no ambiguity in any dimension – the participant can remove all ambiguity.

To explore a more general setup in which ambiguity appears in multiple different dimensions, consider the following variations on the classical Ellsberg experiment. Participants face the same urn as above, and as before only a single chip will be drawn, and the participant is asked to guess a color. In one variation, the participants is paid only if a black chip is drawn, but she is paid a number of dollars equal to the number of chips in the urn of a color chosen: if X is the number of chips in the urn that have the color chosen by the participant, he wins $\$X$ if a black chip is drawn. In this case the probability of winning is not ambiguous – as the lotteries is paid only if a black chip is drawn – but the amount won is. In another variation, the participant is again asked to choose a color, and she is paid, if a chip of *that* color is extracted, a number of dollars equal to the number of chips of that color in the urn. In this variation, there is a sense in which the ambiguity is on "two dimensions:" not only the likelihood of winning, but also the amount won. In yet another variation, if the participant guesses correctly, he wins $\$X$, but these are paid X days from the date of the experiment. Here, we have added ambiguity on a "third dimension:" how soon the prize is paid.

This variation of the classical Ellsberg experiment allows us to answer three classes of questions. A first set of questions is concerned with the correlation of ambiguity attitudes across dimensions. Are participants ambiguity averse also when prizes, and not probabilities, are ambiguous? Or when dates are ambiguous? Moreover, is ambiguity aversion a stable feature 'across dimensions,' i.e. if a participant is ambiguity averse in one dimension, is he more likely to be ambiguity averse in another?

A second set of questions is concerned with situations in which multiple dimensions can be ambiguous at the same time, and where correlations may exist between ambiguous variables across different dimensions. How is an individual's willingness to accept ambiguity in one dimension depends on the presence of ambiguity in other dimensions? For example, suppose an individual could decide only whether or not the probability of winning will be ambiguous. How is her decision affected by whether or not the amount of winning is ambiguous as well? How is it affected by whether or not the date of payment is ambiguous? And if it affected, does the agent chooses to opt for the more ambiguous option in which the various dimensions are positively correlated, or not? These questions have a particular relevance because in real life most often we face options which involve ambiguities in many dimensions at the same time, some of which are fixed and cannot be completely removed.

The third and last set of questions explore whether participants prefer options with no ambiguity to options in which two or more dimensions are ambiguous. Are participants still ambiguity averse? Is the proportion of participants who are averse to "multi-dimensional" ambiguity higher or lower than the proportion who are averse to each form of "single-dimensional" ambiguity?

This paper addresses these questions with an experimental design that adopts the above variation of the canonical Ellsberg framework. A key feature of this framework is that there is a *single* source of ambiguity, namely, the distribution of colors in an urn with a known number of chips. This is crucial for our analysis: because there is only one source of ambiguity, any difference in behavior cannot be due to differences in beliefs.

One of the reasons for using experiments to explore the questions above is that existing theoretical models make little to no predictions over which behavior should agents follow in these situations (see Section 5). This flexibility/lack of predictive power is desirable if there is a large heterogeneity of behavior in the population – we need a flexible model to capture it. If, however, there exists some systematic, representative behavior that individuals are more likely to exhibit in these environment, then we would instead like to have models that capture this pattern – models with a higher predictive power. Identifying such regularities is then a first necessary step to understand which additional restrictions should be imposed on existing models.[1]

Our main findings are the following:

1. *The majority of participants are averse to having ambiguity in only one dimension ("single-dimensional" ambiguity), regardless of what that dimension is* (probability/prize/time). The proportions, however, vary across the different dimensions. Furthermore, the sets of participants who are ambiguity averse in the various dimensions are, with some degree of error, nested: the largest set includes participants who are averse only to ambiguous prizes (82%); participants who are averse to ambiguous probabilities tend to be a subset of it (76%); and the set of participants who is averse to ambiguous dates is then a subset of both (52%).

2. *When one dimension is fixed and ambiguous in all available options, and participants can choose whether or not to have ambiguity in another dimension, the majority of participants prefer ambiguity in both dimensions. Moreover, participants tend to choose the option in which the ambiguity in the dimension they can affect is perfectly correlated with the ambiguity in the fixed dimension.* That is, participants who preferred no ambiguity when that option was available, tend to prefer the gamble with the "*most exposure*" to the ambiguous variable, when (at least) one of the gamble's dimensions is ambiguous and fixed

[1] To better illustrate, it is helpful to make the analogy to the experimental literature on repeated games. The various folk theorems establish that "almost anything" can be supported in equilibrium. But do individuals who actually engage in indefinite repeated interaction exhibit such heterogeneity of behavior? This question has given rise to a growing experimental literature that, like us, investigates whether actual behavior tends to systematically select one of the predicted behaviors. These findings have in turns led to the development of various refinements meant to increase the predictive power of the theories.

in all the available gambles. To illustrate, consider first the standard Ellsberg gamble in which a participant is paid \$20 at the end of the experiment, if he correctly guesses the color of the chip to be drawn. About 76% of participants bet on black, and only 12% bet on green. However, when the amount of the prize is fixed at \$$g$ (the number of green chips in the urn), then only 33% bet on black, while 55% bet on green. Similarly, consider the gamble where a participant chooses a color X, and he wins \$20 only if a black chip comes out, but the payment is made in x days, where x is the number of chips of the color X. About 51% of participants choose black and only 16% choose green. In contrast, when the participant wins \$$r$ (the number of red chips) only if the color red comes up, only 30% of participants choose black, while 48% choose green (notice that the prize amount is perfectly correlated with how quickly the payment is received).[2]

3. *When comparing options with ambiguity in multiple dimensions against options with no ambiguity, the majority of participants prefers the option with no ambiguity.* However, the proportion of participants who prefer no ambiguity to ambiguity in multiple dimensions tends to be smaller than the proportion preferring no ambiguity to ambiguity in a single dimension. For instance, 82% of participants are averse to ambiguity only in prizes, 76% are averse to ambiguity only in probability, but only 67% prefer the unambiguous option to an option in which both prize and probabilities are ambiguous and perfectly correlated.

Our findings therefore suggest that, despite the richness of behavior that is consistent with our models, individuals tend to exhibit one particular pattern of behavior: aversion to "single-dimensional" ambiguity, (milder) aversion to "multi-dimensional" ambiguity; and a preference for "multi-dimensional" ambiguity over "single-dimensional" ambiguity. As a robustness check, we also run an additional experiment in a very different participant pool, Amazon's Mechanical Turk, and we find a very similar pattern (see Section 4).

It is not our intention to present the above findings as "surprising," in the sense that they contradict the intuitions we have from our models: as we mentioned above, current model do allow for the observed behavior. At the same time, however, as these models make essentially no predictions on what should participants choose in

[2]All the changes described are significant at the 1% level.

these situations, without actually running an experiment there is no a priori reason to expect that that the majority of participants would exhibit the patterns above. To the best of our knowledge, we are the first to investigate attitudes towards ambiguity in either probability, prize or time, or any combination of these dimensions. The closest study to ours is the recent independent work of Eichberger et al. (2011). These authors investigates the question of whether individuals that are faced with two unrelated and independent sources of ambiguity treat these two sources as independent of each other. This question is addressed with a natural extension of the classical Ellsberg 2-urn experiment where a participant wins prize $x \in \{a, b\}$ if he guesses correctly and $y \in \{a, b\}/\{x\}$ otherwise. They investigate whether the behavior of participants change if they they do not know whether $x = a$ or $x = b$, where this uncertainty is determined independently of the composition of the urn. While both this study and ours incorporates ambiguity in probability and prize, the two papers ask different questions and employ completely different designs. The most important difference lies on the fact that their experiment exploit distinct sources of ambiguity for prize and probabilities – therefore testing whether subject consider them as independent. By contrast, as we mention above in our analysis all ambiguity comes from a unique source, allowing us to concentrate on the different attitudes without having to consider the beliefs on the joint distribution. We, therefore, view both works as providing complementary insights into the complex nature of attitudes towards ambiguity.

Our findings may have potential implications for decision-making under ambiguity in applications. In most concrete economic settings, a decision-maker cannot remove all ambiguity from choice problems he faces. For example, even if the decision-maker could choose a safe investment that guarantees a certain interest, the inflation rate may be ambiguous, there may be uncertainty regarding the decision-maker's need for liquidity (hence the time of payment), *etc.*. Our results suggest that in many circumstances, decision-makers may be more likely to prefer uncertain prospects where more dimensions are ambiguous and correlated.

The rest of the paper is organized as follows. Section 2 describes the experimental design. Section 3 presents the results of the lab experiment. Section 4 compares the lab results to the results of an online experimental treatment conducted via Amazon's Mechanical Turk. Section 5 illustrates how existing models can accommodate a rich pattern of behavior. Concluding remarks are given in Section 6. The Appendix contains supplementary analysis as well as a description of the instructions and screen shots of the experiment.

2 Experimental design

The lab experiment was carried out in the Social Science Experimental Laboratory (SSEL) at the California Institute of Technology. Participants were undergraduate students at Caltech, recruited from a pool of volunteer participants, maintained by SSEL. There were a total of 97 participants. There were a total of four sessions, each lasting about 30 minutes, and each participant took part in one and only one session.

All sessions used the following procedure. The participants read instructions that were printed on paper and which they could refer to at any point in the experiment (a copy of the instructions appears in Appendix C). The experimenter stood in front of the participants and showed them an opaque cloth bag. The participants were told that the bag contains 60 colored poker chips. Of these 20 are black, while each of the remaining 40 chips can have one of two colors: red or green. Let r denote the number of red chips in the bag and let g denote the number of green chips. The participants know neither the values of r and g, nor how these values are determined. They were told that they could inspect the content of the bag at the end of the experiment.

At the end of the experiment a single chip will be drawn from the bag, and the participants' task is to make choices between groups of two or three gambles that depend on the composition of the bag and on the color of the chip extracted. A gamble assigns to each color that could be extracted an amount of dollars m and a time of payment t. For simplicity of exposition, in what follows we shall use the following notation. Given the (ambiguous) composition of the bag, the probability of extracting a a black, red, or green chip are, respectively, $\frac{20}{60}$, $\frac{r}{60}$, $\frac{g}{60}$. We then denote gambles by a triplets $(p, \$m, t)$, by which we understand the gamble that pays $\$m$ in t days with probability p, and \$0 otherwise.[3] All the three dimensions of a gamble could depend on the composition of the bag: for example, $(\frac{r}{60}, \$g, r \text{ days})$ denotes the gambles that pays $\$g$ in r days with probability $\frac{r}{60}$, i.e. if a Red chip is extracted.

Participants made their choices on a computer. The decision tasks were divided into seven screens. Five of the screens displayed nine gambles arranged in three rows

[3] Each gamble was presented as 3×2 table, where each row corresponded to a color and the two columns represented the payment-date pair (see Figure 1). Moreover, since the term "gamble" may have a negative connotation, we used the term "lottery" in the experiment.

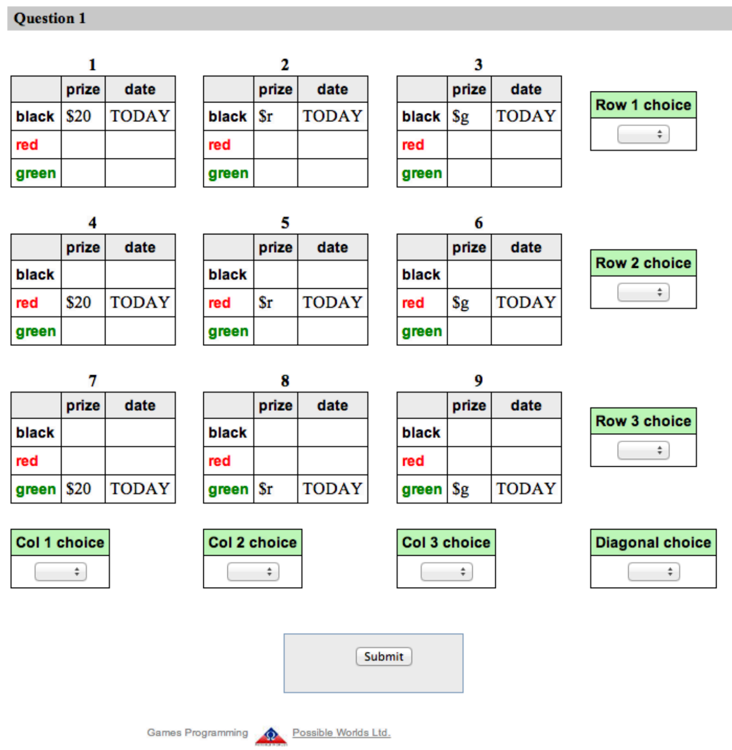

Figure 1: Screenshot of the typical screen

of three gambles; two screens had only six gambles arranged in two rows of three gambles. In each screen, for each row/column a participant was asked to choose his most preferred gamble in the row/column. For screens with nine gambles, a participant was also asked to choose his most preferred gamble on the main diagonal (from top-left to bottom-right).[4] Figure 1 displays screen-shot of one of the seven screens (online Appendix D contains the remaining six screen-shots).

The choice problems in these screens can be categorized as follows.

1. *Attitude toward "ambiguity in probability"*. The participant chooses the probability of winning from $\{\frac{20}{60}, \frac{r}{60}, \frac{g}{60}\}$, for gambles in which the amount of winning is fixed at x dollars and the date of payment is fixed at t days away, where (x, t)

[4]Hence, five screens consisted of seven choice problems (with three alternatives each), while two screens consisted of five choice problems (three of which had two alternatives and the remaining two had three alternatives).

included the following values:

$$\begin{array}{lcccccccc} x: & 20 & 20 & r & g & r & g & 20 & 20 \\ t: & 0 & 20 & 0 & 0 & 20 & 20 & r & g \end{array}$$

2. *Attitude toward "ambiguity in prizes".* The participant chooses the dollar amount of winnings from $\{20, r, g\}$, when the probability of winning is fixed at p and the date of payment is fixed at t days away, where (p, t) included the following values:

$$\begin{array}{lccccccccc} p: & \frac{20}{60} & \frac{20}{60} & \frac{20}{60} & \frac{20}{60} & 1 & \frac{r}{60} & \frac{g}{60} & \frac{r}{60} & \frac{g}{60} \\ t: & 0 & 20 & r & g & 20 & 0 & 0 & 20 & 20 \end{array}$$

3. *Attitude toward "ambiguity in timing".* The participant chooses whether payments will be made in 20 days, r days or g days, when the amount of winning is fixed at x dollars and the probability of payment is fixed at p, where (p, x) included the following values:

$$\begin{array}{lccccccc} p: & 1 & \frac{20}{60} & \frac{20}{60} & \frac{r}{60} & \frac{g}{60} & \frac{r}{60} & \frac{g}{60} \\ t: & 20 & r & g & 20 & 20 & r & g \end{array}$$

4. *Attitude towards ambiguity in "two dimensions".* Each dimension may be viewed as having a value measured in units appropriate to that dimension: each unit in the probability dimension is $\frac{1}{60}$, each unit in the prize dimension is \$1, and each unit in the time dimension is one day. The participant chooses $x \in \{20, r, g\}$ such that the values of two dimensions is x units, while the value of the third dimension is fixed and non-ambiguous (and zero when it is the date).

5. *Attitude towards ambiguity in "all three dimensions".* The participant chooses $x \in \{20, r, g\}$ such that the value of each dimension is x units.

After the instructions were read and any questions were answered, the participants made their choices on a computer. All participants started with the screen in which all payments were made at the end of the experiment (the one depicted in Figure 1). This screen contained the standard Ellsberg question. Following this screen, each participant was assigned to a random sequence of the six other screens.[5] All participants completed a total of 45 choice problems. At the end of the experiment, for each participant the program randomly selected one of the 45 choice problem, and then the participants witnessed the experimenter draw a chip from the bag and show the content of the bag.[6] This determines the amount won by the participants given the selected question and their choice in that question.

All participants received a show-up fee of $10 at the end of the experiment. Any participant who won additional amount to be paid with no delay received his payment also at the end of the experiment. Participants who earned additional amounts that were to be paid at a later date were given the choice between three methods of delivery: personally picking up a cash from a staff member; receiving the check by mail; or being paid through Paypal (see the instructions in Appendix C for more detail).[7] Of note, a large majority of participants choose to pick up their payment in person from a staff member.

There are many possible ways of displaying the choice problems we are interested in. Our choice of display is motivated by the following considerations. First, we wanted to minimize the inconsistencies that are due to errors or carelessness. If participants were presented with just a vertical list of choice problems, they are more likely to miss inconsistencies in their answers. Our display gives participants a better opportunity to internalize the relationship between different choice problems that have common elements. Similarly, we wanted to minimize the possibility that the effect on choice of ambiguity in a fixed dimension (in the sense that the participant's decision cannot remove this ambiguity) could be due to mistake or inattention. The matrix display emphasizes the multi-dimensionality of the problem, thus increasing the likelihood that participants would understand that they can remove ambiguity in

[5]Only after a participant completed making all choices in a screen, could he press a button to take him to the next screen. Once a participant left a screen, he could not return to it.

[6]Our lab design therefore uses the common Random Decision Selection (RDS) mechanism in which participants are paid for only one randomly selected decision. Assuming only that participants' preferences for gambles respect monotonicity (dominated gambles are never chosen), Azrieli et al. (2012) show that the RDS mechanism is incentive compatible.

[7]Note that the choice problems in our experiment had the feature that whenever a participant had the option to choose an ambiguous date of payment, the non-ambiguous date was a known delay of 20 days. Hence, it is not crucial for the purpose of this experiment to equate the transaction cost of payment today versus payment in the future. Since participants may differ in their ranking of the various payment methods, we allowed participants to choose their preferred method (rather than impose one of the three). Thus, if we normalize the transaction cost of the most preferred method, then each participant is facing this normalized cost.

some dimensions but not in others.

In all the screens apart for the two screens that tested for risk-attitudes, the first option (left-most or top) corresponded to no-ambiguity in the relevant dimension (i.e., "betting on black") the middle option corresponded to "betting on red,"and the last option corresponded to "betting on green". Since we are interested in the question of whether individuals who are "ambiguity-averse" in probabilities are willing to take on ambiguity in other dimensions, we wanted to make it difficult to switch away from the no-ambiguity decision. Participants, who are inattentive and just want to finish the task, are more likely to just click on the first option in the dropdown menu. Thus, by keeping no-ambiguity to be the first option we minimize the risk of misinterpreting such inattentive participants as being ambiguity-seeking.

Fixing the positions of the red and green options allow us to check whether participants are sensitive to the positive and negative correlations across the ambiguous dimensions net of order effects. For example, consider the problem of guessing what color chip will be drawn, when a correct guess pays $\$r$. As explained later, we will interpret a guess of red as choosing "more exposure to ambiguity,"and guessing green as choosing " to hedge against ambiguity."If we allow the two colors to appear in a different order, it is hard to disentangle whether this is due to order effects. If, instead, we fix the order, and we compare with the identical question in which prize is fixed at $\$g$ and find the same results, then we know it cannot be due to order effects.[8]

3 Results

Our analysis of the data proceeds as follows. We start with a basic analysis of the consistency of choices. Next, we analyze participants' attitude towards ambiguity, when only a single dimension – prize/date/probability – is ambiguous. We then turn to analyze how participants react to the contemporaneous presence of multiple forms of ambiguity. First, we *fix* one dimension to be ambiguous, and investigate whether the attitude towards ambiguity in another dimension is affected. Second, we study the agent's preferences between options with no ambiguity and options with different

[8]Alternatively, one could randomize the order and then check whether the location of the question had a statistically significant effect on choice. The disadvantage of this approach is that there would be fewer observations for each choice, as a choice would involve not only the color chosen but also the location of the color in the display.

forms of ambiguity at the same time.

3.1 Preliminaries

We begin by discussing some basic properties of our data. In terms of risk-attitudes, we label as 'risk-loving' participants who chose a gamble that pays \$40 in 20 days if a black chip is drawn (and zero otherwise) over a gamble that pays \$20 in 20 days if a red or green chip is drawn (and zero otherwise). A participant was labeled 'strictly risk-averse' if he chose a gamble that paid \$13 in 20 days no matter which chip was drawn over a gamble that paid \$40 in 20 days if a black chip was drawn (and zero otherwise). About 68% of all lab participants were *not* risk-loving, and about 44% were strictly risk-averse. Thus, about a quarter of the participants could be labeled as risk-neutral.[9]

Because the experiment is designed to be entirely symmetric with respect to the two ambiguous colors, red and green, with the exception of the order in which options are presented (one color always appears before the other), we should expect participants to treat these colors symmetrically. This seems to be the case in our data. First of all, bets on either color are not significantly different. Second, the behavior in situations in which one dimension is fixed and depends on the number of red chips is essentially identical to the behavior in which it depends on the number of green chips.[10]

We also investigated to what extent participants gave consistent answers. One form of consistency is transitivity in the answers.[11] About 64% of participants gave transitive answers across all questions in the seven screens they faced. A second form of consistency concerns answers to identical questions. One of the questions in the experiment was asked three times, and 71% of the participants gave consistent answers.

[9]The two screens that presented the risk-attitude question also tested whether participants changed their decisions if the date of payment was ambiguous (see screen shots in Appendix D). We found no significant change in the answers.

[10]The only exception to this symmetry is a significant difference between red and green for choosing the date when everything else is fixed: when choosing between $(\frac{20}{60}, \$20, 20)$, $(\frac{20}{60}, \$20, r)$, and $(\frac{20}{60}, \$20, g)$, over twice as many participants choose the second compared to the third option. We see no obvious explanation for this asymmetry, especially since it seems to be confined to this specific question.

[11]We should emphasize that a violation of transitivity in this context need not represent a violation of 'rationality,' but could simply be due the fact that participants are indifferent but break indifferences in different way depending on the questions. This is particularly relevant in the present context, where we expect participants to treat bets on red or green in a similar way.

The color (X) chosen by the participant	*Probability*	X	Black	X	Black	Black
	Amount	\$20	\$x	\$20	\$x	\$20
	Date	0 days	0 days	20 days	20 days	x days
Black*		76%	82%	73%	75%	52%
Red		11%	9%	13%	15%	34%
Green		12%	8%	13%	9%	14%

Table 1: Attitude towards uni-dimensional ambiguity. The '*' denotes the non-ambiguous option

This consistency is much stronger in aggregate: essentially identical proportions apply to the three questions. Finally, we verified that most participants gave the same answers to questions that differed only in the color in which the ambiguity is fixed.[12]

In what follows we report our results for the entire participant pool. Results are qualitatively, and in most cases also quantitatively very similar if we restrict attention to only transitive participants.

3.2 Attitude towards ambiguity in each dimension

We now turn to investigate how participants approach ambiguity when only a single dimension of the gamble is ambiguous. In what follows we say that a participant is averse to ambiguity in a choice problem if he chooses the gamble with the most objective information. Table 1 summarizes the aggregate results.[13]

The following observation is implied by this table.

Observation 1. *The majority of participants are averse to ambiguity in each dimension. However, the proportions of ambiguity averse participants vary across dimensions. In particular, the three dimensions can be ranked according to the proportion*

[12]We also checked whether the distribution of responses for each of question in a matrix is independent of the position that the matrix appears in (2nd, ..., 7th) by running 38 Fisher exact tests. None of these tests were significant at the 1% level, and only three (out of the 38) were significant at the 5% level.

[13]The question in which participants are asked their attitude towards ambiguous dates, i.e. to choose between $(\frac{20}{60}, \$20, 20)$, $(\frac{20}{60}, \$20, r)$, and $(\frac{20}{60}, \$20, g)$, was actually asked three times during the lab experiment. Table 1 contains the average answer. Answers are very consistent: the fraction of participants choosing $(\frac{20}{60}, \$20, 20)$ ranges between 51% and 55%, the fraction choosing $(\frac{20}{60}, \$20, r)$ between 33% and 36%, and the fraction choosing $(\frac{20}{60}, \$20, g)$ between 12% and 16%. By analyzing individual questions, 71% of participants exhibit the same attitude in all three questions (the remaining 29% of participants might have given different answers because they were indifferent and broke the indifference in a different way at least in one of the three questions).

of participants who are averse to ambiguity in a single dimension: date $\leq$ probability $\leq$ prize.

Observation 1 is based only on aggregate data. This raises the question of whether, at an individual level, the set of individuals who are averse to single-dimensional ambiguity expands as we change the ambiguous dimension from dates to probability to prize. This is approximately true: between 89% and 92% of participants who are ambiguity averse over dates are also ambiguity averse over prizes, and between 84% and 86% are also ambiguity averse over probabilities;[14] about 91% (67/74) of participants who are ambiguity averse over probabilities are also ambiguity averse over prizes. This suggests that, with some degree of error, we can partition our dataset into four groups: those participants who are not ambiguity averse to any dimensions; those participants who are ambiguity averse only over prizes; participants who are averse to ambiguity in prizes and probabilities; and participants who are averse to all ambiguities. The last group contains at most 52% of the pool.

The relatively high proportion of aversion to ambiguity (only) in prizes may be explained by risk-aversion. For example, a decision maker who is ambiguity *neutral* and follows expected utility would still dislike ambiguity over prizes just by risk aversion.[15] If we redo Table 1 for risk-averse participants, we obtain very similar proportions. Redoing it for risk-loving participants yields Table 2. If we compare with Table 1, the proportion of participants who are averse to ambiguity in probability remains the same, but the proportions of participants who are averse to ambiguity in prizes decreases.

Our data is split 50-50 between participants who are averse to ambiguity (only) in the date to those who are not. One possible explanation for this is that there is heterogeneity in the agent's patience and beliefs on the composition of the bag. Under the assumption that participants' preferences are additively separable over time with a standard exponential discount factor δ, in Section 5 we show that the following is true: for a given prior belief on the number of red chips r in the bag, there exists

[14] In the case of aversion towards ambiguous dates we have a range of answers because, as we mentioned above, the question over ambiguous dates was asked three times. More precisely, the fraction of subjects who are ambiguity averse over prizes amongst those who are ambiguity averse over dates is: 43/49, 45/49, and 47/53; fraction of subjects who are ambiguity averse over probabilities amongst those who are ambiguity averse over dates is: 42/49, 41/49, and 45/53.

[15] We also checked whether winning for sure affects the aversion to ambiguity on the prize (i.e. we asked participants to choose between a sure prize of \$20, a sure prize of \$$r$ and sure prize of \$$g$). We found no significant difference.

The color (X) chosen by the participant	*Probability*	X	Black	X	Black	Black
	Amount	$20	$x	$20	$x	$20
	Date	0 days	0 days	20 days	20 days	x days
Black*		74%	68%	74%	55%	48%
Red		10%	19%	10%	32%	33%
Green		16%	13%	16%	13%	18%

Table 2: Attitude towards uni-dimensional ambiguity for risk-loving participants. The '*' denotes the non-ambiguous option

a threshold discount factor δ^* such that the non-ambiguous gamble is preferred to a gamble in which only the payment date is ambiguous if and only if the discount factor is at least δ^* (where the value of the threshold δ^* depends on the prior belief). That is, even allowing subjects to hold 'pessimistic,' if they are impatient enough they would prefer the ambiguous gamble: intuitively, this is due to the fact that exponential discounting is convex, inducing a preference for variance. Therefore, our data suggest that about half of our subjects are impatient enough to prefer the gamble with an ambiguous payment date.[16]

3.3 'Separability' in the dimensions of ambiguity

We now turn to analyze whether the dimensions of ambiguity are 'separable' in the sense that the presence of a (fixed) ambiguity in one dimension affects the ambiguity attitude of the agent in another dimension. To illustrate, let us compare two choice situations in which the agent chooses the color to bet on: 1) the standard Ellsberg question, in which the agent chooses between $(\frac{20}{60}, \$20, 0)$, $(\frac{r}{60}, \$20, 0)$, and $(\frac{g}{60}, \$20, 0)$; and the identical question in which the prize is fixed at \$r instead of \$20, i.e. the choice between $(\frac{20}{60}, \$r, 0)$, $(\frac{r}{60}, \$r, 0)$, and $(\frac{g}{60}, \$r, 0)$. Will the decision maker choose the same color in the two decision problems? If he does, we say that he exhibits a 'separable' attitude towards ambiguity in probability.

We do not consider this separability neither apriori obvious nor normatively desirable. For example, an agent who chooses to bet on Black in the first (Ellsberg)

[16]Note that even though the time horizon in our experiment is at most 40 days, past experiments have shown that lab participants tend to exhibit abnormally low discount factors (see Camerer (1995)). Hence, there is no apriori reason to expect most participants in the lab experiments to necessarily exhibit aversion to ambiguity in the payment date.

problem might decide to bet on green in the second one, because he is already 'exposed' to the number of red chips via the ambiguity in prizes, and may prefer to 'hedge,' or 'reduce her exposure,' by betting on green: if there are few red chips he might win a small amount, but at least he wins with a high probability.[17] Alternatively, this individual might instead decide to bet on red, to 'increase her exposure:' if there are many red chips he wins a large sum with a large probability, while if there are few red chips he wins a small amount with a small probability. In particular, we say that an individual 'switches to more exposure' if he bets on red (green) in a decision problem where the prize is $\$r$ ($\$g$) and the date is unambiguous, but he bets on black when the prize is \$20 and the date is unambiguous.

The notions of separability, hedging, and more exposure can naturally be generalized to all of our questions. Notice however the following: when the date is unambiguous more exposure means that the participant chooses a gamble where the prize and probability both depend positively on the same color; by contrast, when the date is ambiguous, more exposure instead means that the date is negatively correlated with either the prize or the winning probability. For example, if the winning probability is fixed at $\frac{20}{60}$ and prize is fixed at $\$r$, then the participant hedges if he chooses to be paid in r days, while he opts for more exposure if he chooses to be paid in g days: with the latter bet, when r is high the prize is high and and is paid soon (g is small), and when r is low the prize is low and it is delayed.

Table 3 displays the choices of the agent about ambiguity in probabilities, prize, and date, both when all other dimensions are known, and when they are fixed and ambiguous.

These data reveals a clear pattern of choice.

Observation 2. *Compared to a choice problem that includes a gamble with no ambiguity (and where two of the three dimensions are fixed and unambiguous), making one of the dimensions ambiguous (but fixed), leads to a significant change in behavior, most of which is in the direction of more exposure.*

To illustrate this observation note that 76% of the participants exhibited the standard Ellsberg pattern of betting on black when the prize is \$20 and the payment

[17]To see why a participant may decide to 'hedge', assume he has max-min expected utility preferences (see Section 5) with linear utility and only two priors: probability one on $r = 1$ and probability one on $r = 39$. If he chooses either $(\frac{r}{60}, r, 0)$ or $(\frac{g}{60}, g, 0)$, his expected utility is $\frac{1}{60}$. If, however, he chooses $(\frac{r}{60}, g, 0)$ or $(\frac{g}{60}, r, 0)$, his expected utility is $\frac{39}{60}$.

Choosing Prob.	
(20/60,$20,0)	76%
(r/60,$20,0)	11%
(g/60,$20,0)	12%
(20/60,$r,0)	31%
(r/60,$r,0)	52%
(g/60,$r,0)	18%
(20/60,$20,20)	73%
(r/60,$20,20)	13%
(g/60,$20,20)	13%
(20/60,$20,r)	49%
(r/60,$20,r)	21%
(g/60,$20,r)	30%

Choosing Prize	
(20/60,$20,0)	82%
(20/60,$r,0)	9%
(20/60,$g,0)	8%
(r/60,$20,0)	36%
(r/60,$r,0)	52%
(r/60,$g,0)	12%
(20/60,$20,20)	75%
(20/60,$r,20)	15%
(20/60,$g,20)	9%
(20/60,$20,r)	63%
(20/60,$r,r)	18%
(20/60,$g,r)	20%

Choosing Date	
(20/60,$20,20)	52%
(20/60,$20,r)	34%
(20/60,$20,g)	14%
(r/60,$20,20)	29%
(r/60,$20,r)	15%
(r/60,$20,g)	56%
(20/60,$r,20)	30%
(20/60,$r,r)	22%
(20/60,$r,g)	48%
(r/60,$r,20)	19%
(r/60,$r,r)	24%
(r/60,$r,g)	58%

Table 3: Effect of a fixed ambiguous dimension (aggregate)

date is today. Only 11% chose to bet on red in this choice problem. However, when the prize is changed to $\$r$ (the number of red chips), more than half of the participants who chose to bet on black (i.e., chose the unambiguous gamble) in the original choice problem, now switch to bet on red (which here means more exposure). Only 28% of the participants bet on black both when the prize is $20 and when it is $\$r$ (i.e., they continue to choose the gamble with the least ambiguity).

A similar effect is observed when *two* dimensions are made ambiguous (see bottom matrix in Table 4). About half of the participants choose a payment date of 20 days over r and g days when a prize of $20 is awarded if a black chip is drawn. When the prize and winning probabilities are changed to $\$r$ and $r/60$, about half of the participants who chose a date of 20 now choose a date of g days, thereby choosing a gamble in which the prize, probability and how soon the payment is made, are all ambiguous and perfectly correlated. In fact, close to 60% of all participants choose this "triple-ambiguity" gamble.

The only case in which making one of the fixed dimensions ambiguous has a small effect is when subject decide on the prize – $20, $\$r$ or $\$g$ – and the date turns ambiguous. In this case 75% of participants choose a prize of $20 when the payment date is 20 days away, while 63% choose it when it is r days away. We cannot reject the hypothesis that the distribution of answers across the two choice problems is the same (in all other cases, we can reject this hypothesis at the 1% level). Indeed, according to Table 4 making the date ambiguous appears to have the smallest impact on choices relative to making the prize or probability ambiguous.

While Table 3 only shows us the effects of fixing one dimension ambiguous and equal to the number of red chips, in the experiment we also ask the corresponding questions in which the fixed ambiguous dimension is equal to the number of green chips. Results are remarkably similar, confirming the robustness of the findings: see Table A.1 in Appendix A. In fact, almost every participant who switches from betting on black, when one of the fixed dimensions is positively correlated with the number of red chips, also tends to switch *in the same direction* (i.e. towards more exposure or hedging) when the same fixed dimension is positively correlated with the number of greens. (This is illustrated in Table A.2 in Appendix A.)

We have also analyzed the behavior at an individual level. In particular, if we focus on an agent who has chosen a particular color to affect the ambiguity in one dimension when the other dimensions are known, we can analyze which color he would tend to choose when one other dimensions is fixed and ambiguous. Table 4 contains this data, where the main diagonal in each of these tables displays the fraction of participants who chose the same color in both choice problems. From the table it is clear how participants change the color to bet on in the direction of *more exposure*.

Our data therefore reveals a strong propensity to prefer a gamble in which both the prize and probability are ambiguous and perfectly correlated (the gamble with more exposure) to either a gamble in which these dimensions are negatively correlated (the gamble that offers hedging) or a gamble in which one of these dimensions is objectively known (the gamble with the least amount of ambiguity). We should emphasize that for some prior beliefs an expected utility maximizer – who is not too risk-averse – would exhibit such preferences because of the convexity induced by this uncertainty. This is easiest to see under risk neutrality: in this case, in the computation of the expected utility the unknown probability and prize are multiplied, thus appearing as a square term (convex), leading her to prefer this option to the option with only one kind of ambiguity. Of course, however, if most of our participants were standard subjective expected utility maximizers we would not have observed such high fraction of participants choosing no ambiguity whenever it is present (more on this in the next subsection).

To account for the possibility of random errors, we also estimated the effect of in-

		(20/60, 20, 0)	(r/60, 20, 0)	(g/60, 20, 0)	
Choose PROB. -	(20/60, r, 0)	**.28**	**.02**	**.01**	.31
Effect of fixed	(r/60, r, 0)	**.40**	**.05**	**.06**	.52
ambiguous PRIZE	(g/60, r, 0)	**.08**	**.04**	**.05**	.18
		.76	.11	.12	1

		(20/60, 20, 0)	(r/60, 20, 0)	(g/60, 20, 0)	
Choose PROB. -	(20/60, 20, r)	**.45**	**.02**	**.02**	.49
Effect of fixed	(r/60, 20, r)	**.08**	**.08**	**.04**	.21
ambiguous DATE	(g/60, 20, r)	**.20**	**.03**	**.07**	.30
		.73	.13	.13	1

		(20/60, 20, 0)	(20/60, r, 0)	(20/60, g, 0)	
Choose PRIZE -	(r/60, 20, 0)	**.36**	**.00**	**.00**	.36
Effect of fixed	(r/60, r, 0)	**.41**	**.06**	**.04**	.52
ambiguous PROB.	(r/60, g, 0)	**.05**	**.03**	**.04**	.12
		.82	.09	.08	1

		(20/60, 20, 20)	(20/60, r, 20)	(20/60, g, 20)	
Choose PRIZE -	(20/60, 20, r)	**.62**	**.01**	**.00**	.63
Effect of fixed	(20/60, r, r)	**.08**	**.07**	**.02**	.18
ambiguous DATE	(20/60, g, r)	**.05**	**.07**	**.07**	.20
		.75	.15	.09	1^

		(20/60, 20, 20)	(20/60, 20, r)	(20/60, 20, g)	
Choose DATE -	(r/60, 20, 20)	**.28**	**.00**	**.01**	.29
Effect of fixed	(r/60, 20, r)	**.01**	**.12**	**.02**	.15
ambiguous PROB.	(r/60, 20, g)	**.26**	**.21**	**.09**	.56
		.55	.33	.12	1

		(20/60, 20, 20)	(20/60, 20, r)	(20/60, 20, g)	
Choose DATE -	(20/60, r, 20)	**.27**	**.02**	**.01**	.30
Effect of fixed	(20/60, r, r)	**.07**	**.12**	**.02**	.22
ambiguous PRIZE	(20/60, r, g)	**.16**	**.19**	**.13**	.48
		.51	.33	.16	1

		(20/60, 20, 20)	(20/60, 20, r)	(20/60, 20, g)	
Choose DATE -	(r/60, r, 20)	**.16**	**.02**	**.00**	.19
Effect of fixed	(r/60, r, r)	**.08**	**.11**	**.04**	.24
ambiguous PRIZE	(r/60, r, g)	**.26**	**.23**	**.09**	.58
& ambiguous PROB.		.51	.36	.13	1

^: not significant

Table 4: Effect of a fixed ambiguous dimension (individual)

Choosing Probability	Coeff. (Std. Err.)	RRR
Constant	-1.7987 (0.2773)	
Amb. Prize Red	2.3095*** (0.3486)	10.069
Amb. Prize Green	0.8179** (0.3519)	2.266
Amb. Date Red	0.9232*** (0.2843)	2.517
Amb. Date Green	1.5039 (0.2550)	4.499

Choosing Prize	Coeff. (Std. Err.)	RRR
Constant	-1.7874 (0.2472)	
Amb. Prob. Red	2.1440*** (0.2876)	8.534
Amb. Prob. Green	0.4790 (0.3434)	1.276
Amb. Date Red	0.1779 (0.2311)	2.241
Amb. Date Green	0.7777*** (0.1828)	1.842

Choosing Date	Coeff. (Std. Err.)	RRR
Constant	-0.4308** (0.1855)	
Amb. Prob. Red	-0.1934 (0.2690)	0.824
Amb. Prob. Green	1.0708*** (0.2115)	2.918
Amb. Prize Red	0.1599 (0.2278)	1.173
Amb. Prize Green	0.8812*** (0.1730)	2.414
Amb. Prob. & Prize Red	0.6759** (0.3080)	1.966
Amb. Prob. & Prize Green	1.4173*** (0.2659)	4.126

Table 5: Multivariate Logit regressions on the impact of making fixed dimensions ambiguous when the participants can affect the ambiguity of one dimension. The constant is the log odds ratio between choosing red to choosing black when the remaining dimension is fixed and objectively known. *** and ** denote significance at the 1% and 5% levels, respectively.

troducing fixed ambiguous dimensions using multinomial logistical regressions. Table 5 displays the estimation results for each of the three dimensions when participants could affect the ambiguity in only a single dimension (Table A.3 in Appendix A displays estimation results of linear regressions).[18]

To better understand these, estimates consider the case in which the participant chooses red when the only dimension he can affect is the winning probability. The constant term in this case is -1.8, which is the log of the ratio of participants who choose red when all fixed dimensions are objectively known (0.12) to the ratio of participants who choose black in this case (0.75). When the prize is changed from \$20 to \$$r$, the distribution of choices changes to 31% on black and 52% on red. This corresponds to a log ratio of 0.51, which is 2.31 times higher than the constant. This is the coefficient of the variable *Amb. Prize Red*, which is positive and significant (at

[18]These regressions estimate the log ratio of choosing the ambiguous color red relative to the non-ambiguous color black. The constant term estimates this log ratio in the benchmark case where the fixed dimensions are non-ambiguous (e.g. \$20 and 20 days when participant chooses probability). Table 5 reports both the coefficients and the relative risk ratio. (The relative-risk ratio is the exponential of the regression coefficient.)

1%), reflecting the fact that the log odds ratio of choosing red has gone up when the winning probability was changed from 20/60 to $r/60$. These findings support our conclusions from the above analysis of individual behavior. Tables A.4 and A.5 in Appendix A display the regression analysis for choosing the color green, which is essentially identical, and for the remaining cases that were covered by Table 3 above. These regressions also support our above conclusions for the propensity to choose more exposure.

Finally, we should emphasize that we can interpret bets as having more or less exposure to uncertainty, in a way that is independent of the participant's beliefs, only because in our experiment all ambiguity – whether it is on probability, prize, date or any combination of these dimensions – is determined by the *same random variable*: the number of red (green) chips in the urn. To understand the importance of this, consider an alternative environment where each dimension of the gamble is determined by a different random variable. For example, suppose there were two separate urns with 60 chips each, both of which contain 20 black chips, but with possibly different distribution of red and green. Now consider a bet which pays if a given color is drawn from one urn, but the monetary prize (to be paid immediately) equals the number of red chips in the second urn. In this example, whether betting on red on the first urn means more or less exposure to uncertainty depends on the participant's beliefs regarding the *joint* distribution over the composition of the two urns (e.g. a participant may believe that the number of red chips in one urn equals the number of green chips in the other, or that the two urns are identical).[19]

3.4 Attitude towards compound ambiguity

We now turn to analyze the case in which participants face the choice between no ambiguity and gambles having ambiguity in more than one dimension. While our analysis in the previous section shows that participants might prefer to have multiple dimensions being ambiguous to only one dimensions being ambiguous, we do not know how these compare with options with no ambiguity. Table 6 displays this comparison.

This table implies the following.

Observation 3. *The majority of participants choose the non-ambiguous gamble over*

[19]This is similar to the situation studied in Eichberger et al. (2011).

Amb. in Prob & Prize	
(20/60, $20, 20)	64%
(r/60, $r, 20)	21%
(g/60, $g, 20)	15%

Amb. in Prize & Date	
(20/60, $20, 20)	77%
(20/60, $r, r)	15%
(20/60, $g, g)	7%

Amb. in Prob & Date	
(20/60, $20, 20)	71%
(r/60, $20, r)	19%
(g/60, $20, g)	10%

Amb. in Prob, Prize & Date	
(20/60, $20, 20)	78%
(r/60, $r, r)	14%
(g/60, $g, g)	7%

Table 6: Choice between no-ambiguity and multi-dimensional ambiguity.

gambles in which multiple dimensions are ambiguous.

Observation 3 above shows that, even though we have seen that a large fraction of participants are attracted to an ambiguous gamble where the prize is perfectly correlated with the winning probability, participants still prefer options with no ambiguity. Consider for example the gamble that pays $\$r$ today if a red chip comes up. About 52% of participants choose this gamble when they can only affect the *winning probability* while the prize and date are fixed at $\$r$ and zero days, respectively. Exactly the same percentage chooses this gamble when participants can only affect the *prize* while the winning probability and date are fixed at $r/60$ and zero days, respectively. However, the proportion choosing $(\frac{r}{60}, \$r, 0)$ drops to 18% when the non-ambiguous gamble $(\frac{20}{60}, \$20, 0)$ is also available.

Even if the majority of participants prefer the option with no ambiguity, if we compare Table 6 with Table 1, we observe that the fraction of participants who choose the no-ambiguity gamble $(\frac{20}{60}, \$20, 20)$ drops from 75% to 64% when $(\frac{20}{60}, \$r, 20)$ and $(\frac{20}{60}, \$g, 20)$ are replaced with $(\frac{r}{60}, \$r, 20)$ and $(\frac{g}{60}, \$g, 20)$. Instead, when the time dimension becomes also ambiguous – i.e. when $(\frac{r}{60}, \$r, 20)$ and $(\frac{g}{60}, \$g, 20)$ are replaced with $(\frac{r}{60}, \$r, r)$ and $(\frac{g}{60}, \$g, g)$ – the percentage of participants who opt for no ambiguity rises to almost 80%.

As the next observation shows, the behavioral patterns observed at the aggregate level are confirmed at the individual level.

Observation 4. *Consider the set of participants who (i) choose no ambiguity when they can affect the ambiguity in only a single dimension, while the remaining dimensions are objectively known, but (ii) choose an ambiguous gamble with more exposure* *when one of the fixed objective dimensions becomes ambiguous. Most of these participants prefer no ambiguity to an ambiguous gamble with more exposure.*

In particular, the only situation in which multiple-ambiguity-with-more-exposure is available alongside no-ambiguity is when there is ambiguity in probability and prize. When choosing prizes, there are 40 participants (out of 97) who "switch" from the known color to the more ambiguous one with more exposure when the probability becomes fixed at $r/60$. Of these, however, 28 choose no-ambiguity to multiple-ambiguity-with-more-exposure when both are available. (Similarly, 39 switch when the probability is fixed at $g/60$, and 29 of them choose no-ambiguity over multiple-ambiguity.) When choosing probabilities, there are 40 participants who switch from the known color to the ambiguous one with more exposure when the prize becomes fixed at $\$r$. Of these, 31 choose no-ambiguity over multiple-ambiguity-with-more-exposure when both are available. (Similarly, 39 "switch" when the prize is fixed at $\$g$, and 29 of them choose no-ambiguity over multiple-ambiguity.)

To take a systematic account of possible errors, and to better understand the impact of allowing for ambiguity on multiple dimensions, we ran a series of multinomial regressions (linear regressions return essentially identical results.) The results are summarized in Table 7.[20] Consistently with our observations above, relative to the case in which participants choose either the winning probability or the prize amount (while remaining dimensions are known), allowing for ambiguity in both dimensions significantly lowers the fraction who choose no-ambiguity, while allowing for ambiguity in the date has no significant effect. In contrast, compared to the case in which participants choose only the date, allowing for ambiguity in prize or probability significantly increases the fraction who choose no-ambiguity.

[20] To understand these results, consider the regression which takes choice over probabilities as the benchmark (the top part of Table 7). This regression asks: does allowing participants to choose ambiguity in prizes and dates change their tendency of choosing the non-ambiguous option? When participants choose the winning probability (from 20/60, $r/60$ and $g/60$), while the prize and date are known, about 75% of them (on average) choose no ambiguity (this percentage corresponds to the constant term in the linear regression, while the constant in the multinomial regression is the log of the odds ratio: $log(.748/(1-.748)) = 1.08$.) However, when the prize can also be chosen to be ambiguous, only 65% choose no-ambiguity. Hence, 9.3% fewer subjects choose the non-ambiguous option when they can also control the ambiguity in prizes. This is the coefficient of *Prob & Prize* in the linear regression (because of the type of questions asked, the only options for ambiguity in two dimensions had the same colors for both dimensions). The log odds ratio here is $log(.655/(1-.655)) = .63$, which is .45 less than before. This difference of -.45 gives the logit coefficient of *Prob & Prize*.

	Coefficient	P>z
Choosing Prob.		
Prob & Prize	-0.445	0.025
	(0.198)	
Prob & Date	-0.183	0.445
	(0.240)	
Constant	1.085	0.000
	(0.206)	
Choosing Prize		
Prize & Prob.	-0.627	0.000
	(0.159)	
Prize & Date	0.020	0.879
	(0.133)	
Constant	1.266	0.000
	(0.213)	
Choosing Date		
Date & Prob.	0.840	0.001
	(0.247)	
Date & Prize	1.224	0.000
	(0.230)	
Constant	0.062	0.721
	(0.173)	

Table 7: Effect on choosing no-ambiguity of allowing ambiguity in multiple dimensions.

4 A robustness treatment

A variant of the lab experiment was also conducted on Amazon.com's Mechanical Turk (MT) platform (see https://www.MT.com/MT/welcome).[21] This provided some evidence on the robustness of the findings in the Caltech lab. A total of 355

[21]Mechanical Turk is a "crowdsourcing" Internet marketplace that enables businesses and researchers (known as "Requesters") to post links to decision tasks known as HITs (Human Intelligence Tasks). These tasks can take the form of either multiple choice questions or free-form writing. Workers (called "Providers") can then browse among existing tasks and complete them for a monetary payment set by the Requester. Requesters can ask that Workers fulfill some qualifications before engaging a task, and they can set up a test in order to verify the qualification. They can also accept or reject the result sent by the Worker, which reflects on the Worker's reputation. Currently, Workers can have an address anywhere in the world. Payments for completing tasks can be redeemed on Amazon.com via gift certificate or be later transferred to a Worker's U.S. bank account. Requesters pay 10 percent of the price of successfully completed HITs to Amazon. According to the New York Times, in March 2007, there were reportedly more than 100,000 workers in over 100 countries (Jason Pontin "Artificial Intelligence, With Help From the Humans", published in the New York Times on March 25, 2007). For a summary demographics on MT users see http://www.behind-the-enemy-lines.com/2008/03/mechanical-turk-demographics.html. For an interactive map pinpointing the locations of 50,000 of their MT workers around the world see http://techlist.com/MT/global-MT-worker-map.php.

Choosing Prob.	
(20/60,\$20,0)	74%
(r/60,\$20,0)	17%
(g/60,\$20,0)	10%
(20/60,\$r,0)	54%
(r/60,\$r,0)	34%
(g/60,\$r,0)	12%
(20/60,\$20,20)	73%
(r/60,\$20,20)	18%
(g/60,\$20,20)	9%
(20/60,\$r,20)	48%
(r/60,\$r,20)	26%
(g/60,\$r,20)	26%

Choosing Prize	
(20/60,\$20,0)	90%
(20/60,\$r,0)	9%
(20/60,\$g,0)	1%
(r/60,\$20,0)	69%
(r/60,\$r,0)	22%
(r/60,\$g,0)	8%
(20/60,\$20,20)	86%
(20/60,\$r,20)	8%
(20/60,\$g,20)	6%
(20/60,\$20,r)	74%
(20/60,\$r,r)	20%
(20/60,\$g,r)	6%

Choosing Date	
(20/60,\$20,20)	72%
(20/60,\$20,r)	21%
(20/60,\$20,g)	7%
(r/60,\$20,20)	55%
(r/60,\$20,r)	27%
(r/60,\$20,g)	18%
(20/60,\$r,20)	47%
(20/60,\$r,r)	32%
(20/60,\$r,g)	21%
(r/60,\$r,20)	48%
(r/60,\$r,r)	35%
(r/60,\$r,g)	17%

Table 8: Choice distributions of MT when participants can affect the ambiguity of only a single dimension.

participants participated in the MT treatment,[22] which used the procedure used in the Caltech lab with the following modifications. First, participants read the instructions online and were asked to imagine an opaque bag with colored chips. Second, all choice problems were hypothetical and participants were paid a flat fee of \$1 (participation fees on MT are usually very low and range between \$0.25 to \$2). Third, as MT participants were unlikely to have the patience and attention to answer 45 (repetitive) questions, they were only presented with two screens of problems: the first screen (with payment today) and a second, randomly selected screen.

Overall, the MT data appears to be very similar to the Caltech data.[23] Table 8 shows the results when participants can affect the ambiguity of only a *single* dimension. (All the tables in this subsection focus on the case in which the additional ambiguity is perfectly correlated with the number of red chips. Similar tables for correlation with green appear in Table B.1 Appendix B.)

Table 8 shows that the main patterns exhibited in lab are present in the MT dataset as well. There are, however, some differences. First, the proportion of MT

[22]To minimize the possibility that the same individual participated more than once, participants had to log in with an email address and state that this was their first time participating. A participant who tried to log in more than once with the same email address was refused entry.

[23]MT participants were fairly consistent in their answers: 271 out of 355 (76%) gave transitive answers in the 2 screens they faced.

participants who chose no-ambiguity is weakly higher for each of the dimensions.[24] In addition, the effect of introducing a fixed ambiguous dimension on the choice of ambiguous gambles is milder compared with the Caltech data: relative to the Caltech participants, there is a lower fraction of participants who choose the ambiguous gamble with more exposure.[25] Coherently, the estimates of multinomial regressions (for the effect of making a fixed dimension ambiguous) are qualitatively similar to the estimates obtained from the lab data, but the magnitude of the effect is smaller. (Table B.2 in Appendix B displays the estimation results for the MT data). The distributions of choices in the lab and MT data are even more similar when participants have the option between no-ambiguity and ambiguity in multiple dimensions. (See Table B.3 in Appendix B.) We have also estimated a logit model to study how agents compare options with multiple ambiguities and options with no ambiguities. The estimation results are again very similar to those obtained for the Caltech data (see Table B.4 in Appendix B).[26]

5 Relating the data to theory

We now turn to discuss the theoretical predictions on our dataset made by existing models of decision-making under uncertainty. The main message is that most models have little to no predictive power: most models can accommodate almost *all* possible rankings of the gambles considered in our experiment – both those exhibited by the majority of our participants, but also very different ones. Thus, at least in theory, there is no a priori reason to expect the emergence of a systematic behavior (from two very different pools).

We begin with ambiguity on the prize and/or probability. (We shall discuss ambi-

[24]In contrast, the fraction of risk-loving participants was much higher in the MT sample: 61% compared with 32% in the Caltech data. This relatively high proportion may be due to the fact that the MT questionnaire was hypothetical.

[25]For example, when only the prize can be chosen (\$20, \$$r$ or \$$g$), and the winning probability is changed from 20/60 to $r/60$, the fraction of participants who choose no ambiguity drops from 82% to 36% in Caltech, whereas in MT the drop is from 90% to 69%.

[26]There are only a few notable exceptions. First, relative to the case in which participants can only affect ambiguity in *probability* (*payment date*), there is *no* significant effect on the choice of no-ambiguity when participants can also affect the ambiguity in the *prize* (*winning probability*). Second, relative to the case in which participants can only affect ambiguity in the *prize amount*, there is a *significant negative* effect on the choice of no-ambiguity when participants can also affect the ambiguity in the *winning probability*.

guity in the date separately, as the intuition is different for that case.) In what follows we focus on two well-known models of choice under uncertainty: Max-Min Expected Utility (MMEU, Gilboa and Schmeidler (1989)) and Recursive Non-Expected Utility (RNEU, Segal (1987, 1990)).[27] For simplicity, we impose the standard consistency assumption that any prior must assigns probability 1/3 to a black chip being extracted, and that conditional on there being r red chips in the urn, the probability of drawing a black/red/green chip is $\frac{1}{3}/\frac{r}{60}/\frac{40-r}{60}$. This implies that the prior belief of a participant is defined, effectively, on the number of red chips in the urn (or, equivalently, the number of green chips).

In the MMEU model, a decision-maker has a set of priors Π and a Bernoulli utility function over prizes u. Let $g(x) = (p(x), m(x), 0)$ be a gamble in which the probability and prize are functions of x (which may be constant). According to this model, the value V of a gamble g is the minimal expected utility from the gamble, where the minimum is taken over all the priors in Π. That is, assuming $u(0) = 0$,

$$V(g(x)) = \min_{\pi \in \Pi} \sum_{x=0}^{20} \pi(x) \cdot p(x) \cdot u(m(x)).$$

We assume the decision-maker treats the colors red and green symmetrically: for any prior $\pi \in \Pi$ there exists a prior $\pi' \in \Pi$ s.t. $\pi'(40-r) = \pi(r)$ for $r = 0, 1, \ldots, 40$. It is easy to see that, as long as Π is not a singleton, the decision-maker would exhibit the standard Ellsberg aversion to ambiguity in probability. In addition, if the decision-maker is risk-averse, then he would prefer no-ambiguity to ambiguity in the prize (this is true for any Π).

It is straightforward (but somewhat tedious) to show that if we do not impose further restrictions on Π, then *any* ranking of $(\frac{20}{60}, 20, t)$, $(\frac{x}{60}, x, t)$ and $(\frac{20}{60}, x, t)$ that respects aversion to "single-dimensional ambiguity aversion," i.e. $(\frac{20}{60}, 20, t) \succ (\frac{x}{60}, 20, t)$ and $(\frac{20}{60}, 20, t) \succ (\frac{20}{60}, x, t)$, is compatible with MMEU for some Π and some concave u. Intuitively, this is due to the fact that there are two opposing forces at play when evaluating $(\frac{x}{60}, x, t)$: the more pessimistic the priors in Π, the less the agent will like

[27] We focus on MMEU both for its popularity, and because it is representative of a much larger class of models for which essentially identical arguments could be given. On the other hand, RNEU is a very different model, it has received recent empirical support (Halevy (2007)), and has been recently used to 'explain' some 'paradoxes' that challenge many of the existing models of decision-making under ambiguity (see Machina (2009, 2012), Baillon et al. (2011), Dillenberger and Segal (2012)).

this option; but the higher the variance of these priors, the more he will like it. As discussed in Section 3.3, the latter point follows from the fact that in computing the expected utility of this option (even with a pessimistic prior), the utility and the probability are multiplied by each other, which – if the agent is not too risk averse – generates a convexity that renders this options attractive if variance is high.[28]

Consider now the class of RNEU models. To simplify the exposition, we impose the following restrictions (we will show how a large variety of ranking could be obtained even in this specific case). First, we assume that the agent applies RNEU by applying the Rank-Dependent Utility (RDU) model of Yaari (1987) to first-and second-stage lotteries, with a linear utility u and a *convex*, i.e. pessimistic, weighting function.[29] Second, we assume that belief on r has only two elements in the support: either $20-k$ or $20+k$, with equal probability, where k is an integer between 0 and 20. To describe the corresponding functional let g_i be a gamble in which there is probability p_i of winning m_i (to be paid today) and zero otherwise. Let g_1 and g_2 be two gambles with $m_1 \geq m_2$, and let g be a gamble in which there is 50% of playing g_1 and 50% of playing g_2. According to this model, there exists a utility function u, which we assume to be linear, and a probability weighting function f, which we assume to be convex and satisfying $f(0) = 0$ and $f(1) = 1$, such that the value V assigned g is

$$V(g) = u(m_1) \cdot f(p_1) \cdot f(\frac{1}{2}) + u(m_2) \cdot f(p_2) \cdot [1 - f(\frac{1}{2})]$$

(assuming $u(0) = 0$).

It is easy to see that, as long as f is not linear but convex, this generates the typical Ellsberg ranking of $(\frac{20}{60}, 20, t) \succ (\frac{x}{60}, 20, t)$. Similarly, one can verify for a sufficiently large (small) k, the "full-exposure" gamble $(\frac{x}{60}, x, t)$ is preferred (inferior) to each of the three gambles, $(\frac{20}{60}, 20, t)$, $(\frac{x}{60}, 20, t)$ and $(\frac{20}{60}, x, t)$. The intuition is that accepting the ambiguity of the full exposure gamble, $(\frac{x}{60}, x, t)$, is worthwhile only if one could win a high enough prize with a high enough probability. However, one can find "intermediate" levels of k for which, for some convex f, the agent ranks $(\frac{20}{60}, 20, t)$ above $(\frac{x}{60}, x, t)$, and ranks both above $(\frac{20}{60}, x, t)$ and $(\frac{x}{60}, 20, t)$ – the most

[28] For example, assuming a linear u, we obtain $(\frac{20}{60}, 20, t) \succ (\frac{x}{60}, x, t)$ if Π is the closed convex hull of the set $\{\pi_1, \pi_2\}$, where $\pi_1(10) = 1$ and $\pi_2(30) = 1$. The opposite ranking, $(\frac{x}{60}, x, t) \succ (\frac{20}{60}, 20, t)$, could be obtained if Π is the closed convex hull of the set $\{\pi_3, \pi_4\}$, where $\pi_3(0) = \pi_3(39) = \frac{1}{2}$, and $\pi_4(1) = \pi_4(40) = \frac{1}{2}$.

[29] Indeed the RNEU model is more general: the preferences need not follow RDU, the utility need not be linear, and the probability weighting need not be convex.

common ranking in our experiment. For example, this is true for $f(p) = p^2$ and $k = 10$.[30]

Next, we discuss the theoretical predictions regarding attitudes towards ambiguity in the payment date. Consider the choice between $(\frac{20}{60}, 20, 20)$ and $(\frac{20}{60}, 20, r)$ (a similar argument can be made for a gamble where the date is g days away). Let $\mathbf{p} = (p_{-20}, \ldots, p_{+20})$ be some prior belief on the number of red chips, where p_k is the probability that there are $r = 20 + k$ red chips in the bag. Assume the participants' preferences are additively separable over time with a standard exponential discount factor $\delta \in (0, 1)$. These preferences may be represented by any of the existing decision-theoretic models, as long as this assumption on time preferences is satisfied. Then given the prior $\mathbf{p}$, a participant prefers the non-ambiguous gamble if and only if

$$\sum_{k=-20}^{20} p_k \cdot \delta^{20}(1 - \delta^k) \geq 0 \tag{1}$$

Let $\alpha \equiv -\ln \delta$ and consider an expected utility maximizer with a CARA utility $u(x) = -e^{-\alpha x}$ and wealth zero. This decision-maker would accept a lottery that pays $k \in \{-20, \ldots, 20\}$ with probability p_k if and only if (1) is satisfied. CARA preferences have the property that there exists a threshold risk-aversion coefficient α^* such that any CARA decision-maker with a coefficient $\alpha = \alpha^*$ would be indifferent between accepting and rejecting this lottery, while any decision-maker with a higher (lower) coefficient would reject (accept) it (see Aumann and Serrano (2008)). This implies that any participant with a subjective discount factor $\delta > e^{-\alpha^*}$ would choose the non-ambiguous gamble, and any participant with a lower discount factor would opt for the ambiguous gamble. Hence, sufficiently patient decision-makers would prefer no-ambiguity.

To summarize this section, the following conclusions can be drawn from the decision-theoretic models we considered. The models do generate predictions regarding "single-dimensional" ambiguity: a decision-maker who is averse to ambiguity

[30]Note also that if $f(p) = p^\alpha$ with $\alpha > 1$, then the agent would exhibit the standard Ellsberg paradox (preferring $(\frac{20}{60}, 20, t)$ to $(\frac{x}{60}, 20, t)$) for *any* k. However, if $f(\frac{2}{3}) \cdot f(\frac{1}{2}) > f(\frac{1}{3})$, then for k close to 20 the decision-maker may actually prefer the gamble with the ambiguous probability. At the same time, any such agent would strictly prefer $(\frac{20}{60}, 20, t)$ to $(\frac{20}{60}, x, t)$ for *any* convex f and for *any* k, the no-ambiguity gamble. In this sense, this model accommodates our finding that a participant who was averse to ambiguity in probability also tended to be averse to ambiguity in prizes, but not the converse.

only in the probability dimension (i) would also be averse to ambiguity only in the prize dimension if he is risk-averse, and (ii) would also be averse to ambiguity only in the time dimension if he is sufficiently patient. However, even under reasonable restrictions on the prior beliefs, the models have *no* prediction regarding attitudes towards "multi-dimensional" ambiguity. In particular, they allow a decision-maker who is averse to "single-dimensional" ambiguity to prefer ambiguity in more dimensions.

6 Concluding remarks

The goal of this paper was to explore how individuals actually approach ambiguity when the ambiguity may be in different or in multiple dimensions. While most decision problems in real life involve ambiguity in multiple dimensions, essentially all models in decision-making under uncertainty are agnostic to the distinction between these dimensions, and, in particular, provide no guidance as to what behavior we should expect when individuals choose between gambles that involve in ambiguity in more than one dimension. This paper explores the question of whether there is some systematic, representative behavior that the majority of individuals exhibit.

We address this question by conducting an experiment on two very different participant pools: the lab and Mechanical Turk. The majority of participants in both participant pools exhibit the same systematic behavior: (i) they prefer no ambiguity to ambiguity on any single dimension and to ambiguity on multiple dimensions, and (ii) they prefer "correlated" ambiguity on multiple dimensions to ambiguity on any single dimension.

Our results suggest that in economic settings, where some ambiguity is always present, decision-makers may be more likely to choose uncertain prospects with "more exposure to ambiguity" (in the sense that more dimensions are ambiguous and correlated).

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