Information Effects in Multi-Unit Dutch Auctions

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Abstract This study compares bidding behavior in a multi-unit uniform-price descending price (Dutch) auction under four different information conditions. Bidders are either informed of the number of bidders in the auction, or know that it is one of two possible sizes; they also either know the number of units remaining for sale or are unaware of how many units have been taken by other bidders. We find that revealing group size decreases bids, and therefore revenue, if units remaining are not shown. When group size is unknown the price also falls if the number of units remaining is revealed. The most efficient and largest revenue outcome occurs when bidders are not provided information on either group size or units remaining.

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JEL Classification Numbers: C9, D44, D02

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1 Introduction

Parking is a problem for universities. In 2009, Chapman University used a descending price (Dutch) auction to allocate its most convenient campus parking permits. The primary goal of the auction was to efficiently allocate on-campus parking. In particular, a limited number of reserved parking permits that assigned a specific parking spot to an individual and spots in an on-campus lot at a low permit/slot ratio were sold.[1] Prior to the development and implementation of the Chapman University auction, reserved parking used a posted price mechanism with a long waiting list to get a reserved spot. On-campus parking was priced the same as regular parking and commuters reported frustration with hunting for an open parking space.

The auction process began by announcing the number of permits available for reserved and on-campus parking. Each of the auctions was separate and the reserve auction was conducted first. The auction for the reserved spots began at a price that we estimated was higher than most drivers would be willing to pay and the price was reduced by $20 dollars every 30 minutes (the auction ran from 9am to 9pm daily). Bidders could accept the current listed price and be guaranteed a spot or they could enter the highest price they would be willing to pay (a proxy bid). In addition, in the reserve auction specific spots on campus were selected by the bidders based on the order of their bid. Thus, a descending bid auction was required to get information on the upper part of the demand curve.

In the reserve auction, the clock price[2] stopped when the number of bids equaled the number of permits available.[3] The winners all paid a uniform price equal to the last accepted bid and they were able to choose the location of their preferred parking spot in order starting with the highest revealed bid. For on-campus preferred parking, every winner paid the uniform price equal to the last accepted bid and was granted access to the on-campus lot.

The reserve auctions were held for the 2009/2010 and the 2010/2011 school years. The bids that were received are shown in Figure 1 ordered from highest to lowest.[4] The vertical line in Figure 1 indicates the (inelastic) supply of reserve spots which was 58 in 2009 and 67 in 2010. The available permits were sold to the highest bidders. The price paid for a reserve spot was $630 in 2009 and $590 in 2010. Figures 1 and 2 display the total price paid for permits which includes the regular parking fee that is paid by all commuters in addition to

[1]The reserved parking permits were for the entire year and the on-campus lot was auctioned in each semester.

[2]A Dutch clock auction opens at a high price. The price ticks down in increments; a bid is accepted when the clock price reaches or falls below the bid.

[3]In case of ties at the stop-out price, spaces were awarded based on time priority (those who entered their bid first had higher priority).

[4]The bids in the graph include both accepted and rejected proxy bids.

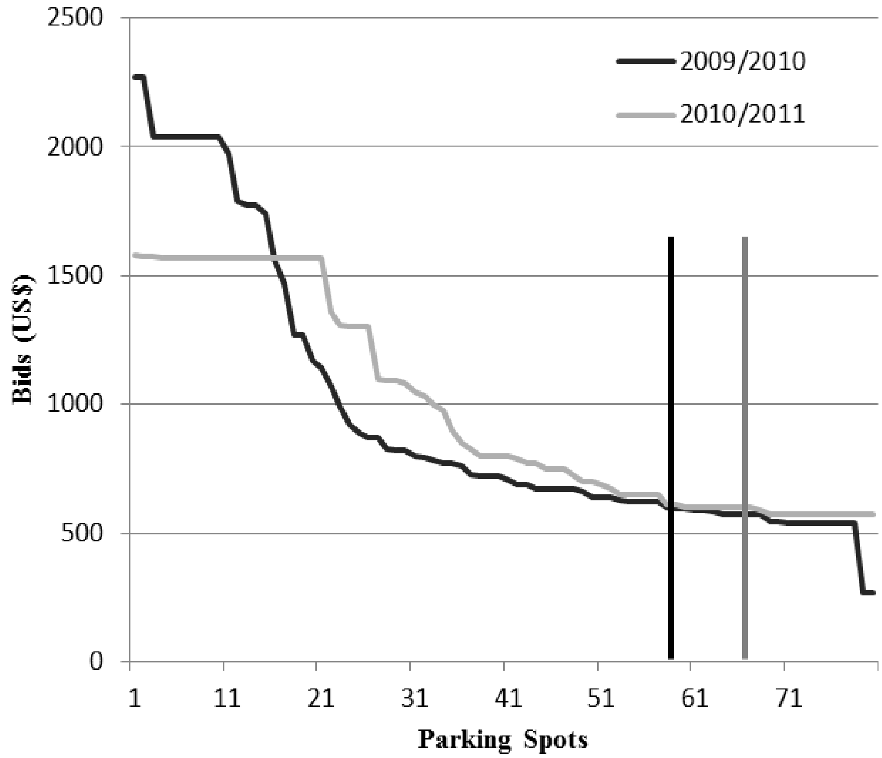

Figure 1: **Reserved Permit Bids**

the auction price.

The bids received for the on-campus parking lot are shown in Figure 2. There were 124 spots auctioned in the Fall of 2009, 185 spots in Spring 2010 and 98 in Fall 2010.[5] The first on-campus auction had lower participation due to a lack of awareness in the community, and Figure 2 shows the growth in participation for the later auctions.

This type of auction raises an important design question about how much information should be provided to bidders. We did not display the number of remaining parking spots as the price dropped. We also did not register all potential bidders before starting which would have provided an upper bound on the potential number of bidders competing for the spots. At the time, how more revelation of information would affect revenue and efficiency was an unanswered empirical question. In this paper we help answer this question by developing the bidding theory under four possible information structures and testing them in the laboratory.

We find in our experiment that revealing more information lowers revenue with independent private values. In the presence of affiliated values, Milgrom and Weber (1982) established the "linkage principle" wherein agents will bid more competitively (higher) when

[5]The number of spots auctioned was different because spots are eliminated in the Fall to accommodate temporary overflow bleachers for the football stadium.

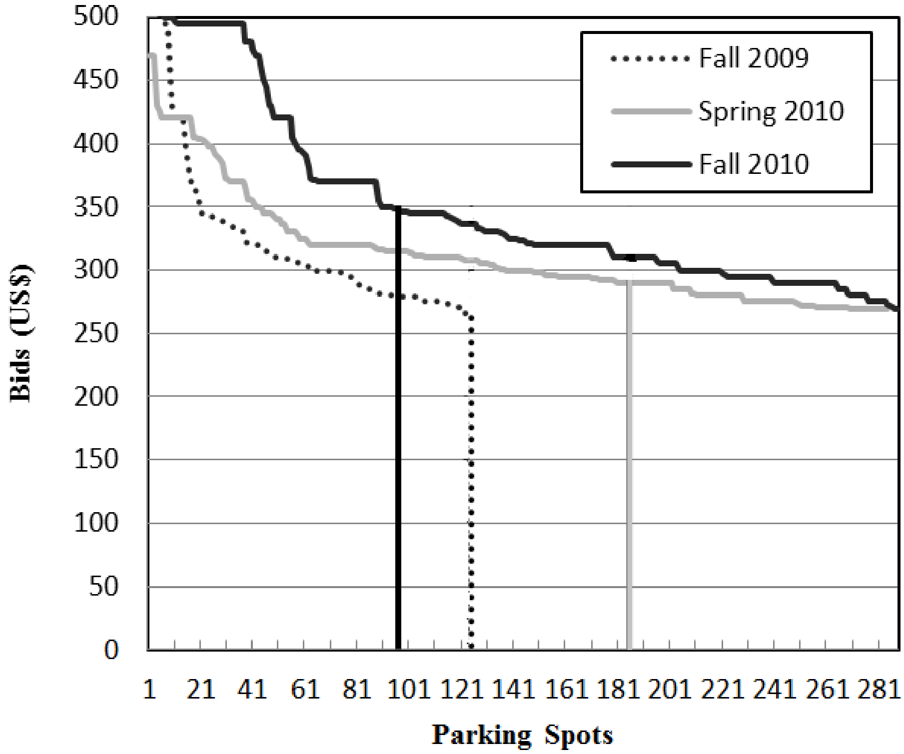

Figure 2: **On-Campus Permit Bids**

more information about values is made public. Perry and Reny (1999) interpret the linkage principle as a generalized guideline for auction design that revenue will be higher when bidders have more information. For the single-unit case, this principle recommends an English auction format over Dutch or sealed-bid formats because participants can learn about the values of other bidders as they see their opponents drop out of the auction. Milgrom and Weber do not address the multi-unit case where bidders gain information about values from the Dutch format when opponents place bids as the price descends. Perry and Reny find that the linkage principle does not generally hold in multi-unit auctions. In their theoretical model, revealing more information about values does not increase revenues in a two-unit Vickrey auction where bidders have values for both items.

McAfee and McMillan (1987) develop a model of the first price auction with private values in which the number of bidders is unknown. They find in their model that the expected revenue is higher when the number of bidders is unknown to participants. Dyer et. al (1989) find that uncertainty about group size raises revenue in single-unit private-value sealed-bid auctions. Our experimental results add to the evidence that revealing group size lowers revenue. Pekec and Tsetlin (2008) provide a comparative institutional analysis of the discriminatory (everyone pays their own bid) and uniform price (everyone pays the first rejected bid) auctions when the number of bidders can be left unknown. They show that, given the level of uncertainty of participation, the discriminatory auction can generate higher revenue than the uniform price auction.

We examine the effects of two kinds of information relevant to multi-unit Dutch auctions - group size and the number of units remaining at any given time. In the following section, we derive the Nash equilibrium bidding strategy for each information structure assuming risk-neutrality. Theoretically, we find that showing units remaining should not affect bidding when group size is known and that disclosure of units should lower bids when group size is unknown. Our experiments confirm these predictions. However, the theory also predicts that showing group size will raise bids on average; however our experiments indicate the opposite is true. We also find that bids are significantly above the Nash prediction in all of our treatments.

From an experimental and market design perspective, environments in which the number of bidders is unknown are commonplace and cause the most deviation from theory. While we have developed the standard theory for the case of a multi-unit Dutch auction, more focus needs to be placed in modeling and experimenting with environments in which the number of bidders is unknown to better match what is the extant world of auctions.

2 Theory

In this section we develop the theory of multiple unit descending price auctions for the various design cases in which the number of units remaining and the number of bidders are known or unknown.

We let m denote the number of units available for sale to $n > m$ buyers that desire at most one unit. Each buyer's value v is drawn from a known and fixed distribution $f(v)$. The draws are i.i.d., and the distribution has the support $[a, b]$.

In the auctions that we analyze, the price p starts at the upper end of the support of the value distribution and falls by e for each tick of the clock. Thus, with z clock ticks, the price is $b - ze$. When the first bidder accepts the current clock price there will be $k = m - 1$ units still available. The process continues until $k = 0$ and the price is set equal to the m^{th} bid. We develop the risk-neutral symmetric Nash equilibrium (RNSNE) bidding strategy $B(v)$ for three of the information cases of this auction. For the fourth case, when bidders know the number of units remaining for sale, but don't know their group size, we use a risk-neutral symmetric Bayes-Nash equilibrium (RNSBNE) strategy because of the Bayesian process agents use to update their beliefs about group size.

2.1 Bidding Strategy If n Is Known (for cases when k is unknown and when k is known)

Vetsikas and Jennings (2010) show that the RNSNE bidding strategy for the m^{th} price auction when values are drawn from a distribution with a cumulative distribution function $F(v)$ when n is common knowledge (and k is unknown) is:

$$\begin{aligned} B(v) &= v - (F(v))^{-(n-m)} \cdot \int_0^v (F(z))^{n-m} dz \\ &= v - v^{-(n-m)} \cdot \frac{v^{n-m+1}}{n-m+1} \\ &= v\frac{n-m}{n-m+1}. \end{aligned} \tag{1}$$

As shown in McCabe, Rassenti, and Smith (1990) the bid function shown in equation (1) also holds when both n and k are common knowledge. This is the multi-unit extension of the single-unit model in Vickrey (1961).

When group size is known, k is not included in the bid function. The intuition is straightforward, agents do not update their bidding strategy based on how many items have been claimed or the downward ticking price because they have already computed the order statistics of the values of their $n-1$ competitors. Seeing the first $m-1$ units get claimed informs the price setter about the values of the highest $m-1$ bidders, but the agent with the m^{th}-highest value who sets the uniform price is not concerned with outbidding the agents with the highest values. When there is one unit remaining, the remaining bidders are essentially in a first price auction with $n-m+1$ bidders against whom they are competing for the last unclaimed item.

2.2 Bidding Strategy If n Is Unknown and k Is Unknown

In this case, we assume that bidders have common knowledge about the possible group sizes in which they will be competing and the probabilities of their occurrence. Let $N = \{n_1, \ldots, n_G\}$ be the vector of the possible group sizes in which buyers could be placed and $P = \{p_1, \ldots, p_G\}$ be the vector of associated probabilities that the current group size is the corresponding value in N.

Assuming a uniform distribution $F'(v) = 1$, the RNSNE bidding strategy for this information structure is (proof can be found in Appendix A):

$$B(v, N, P) = v\frac{\sum_{g=1}^{G} \frac{n_g - m}{n_g - m + 1} p_g v_i^{n_g} \binom{n_g - 1}{m-1}}{\sum_{g=1}^{G} p_g v_i^{n_g} \binom{n_g-1}{m-1}}. \tag{2}$$

The fraction in equation (2) ranges from 1/2 to 1 depending on agents beliefs about their group size. The fraction approaches 1 as each bidder's estimate of their group size increases. When the group size is sufficiently large, agents should bid almost the entire amount of their value to increase their chances of winning an item and making a profit. In Figure 3, we use our experimental parameters to illustrate the difference between the strategies in equations (1) and (2). The solid lines in the figure provide the bid as a function of a bidders' value for known group sizes n of 4 and 8 participants. When n is unknown in our experiment, $N = \{4, 8\}$ and $P = \{.5, .5\}$, meaning bidders know there is a 50% chance that they are in a group of 4 and a 50% chance that they are in a group of 8. In this uncertain group size case, the bid function is given by the dashed curve that lies between the known group size cases.

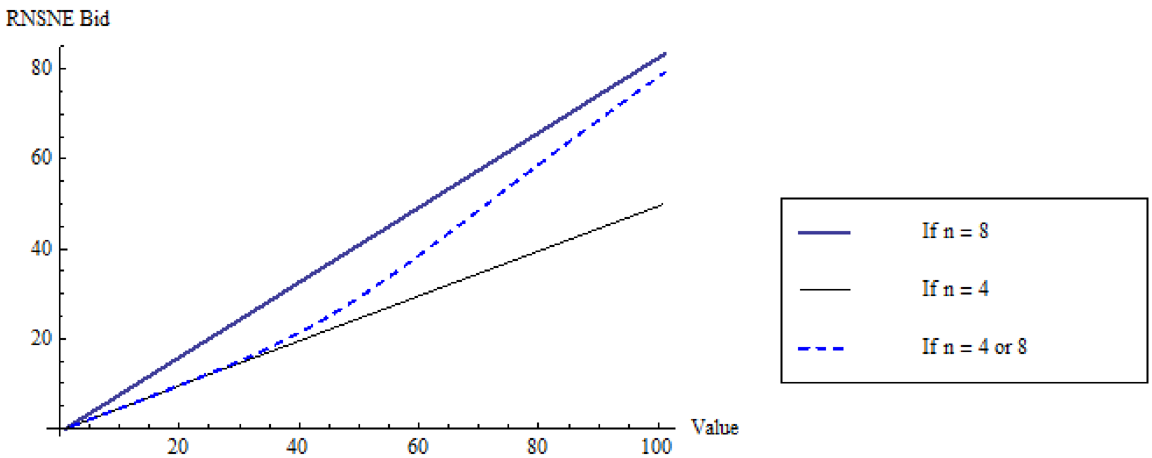

Figure 3: **RNSNE Bidding Strategy as a Function of Value**

2.3 Bidding Strategy If n Is Unknown and k Is Known

The bidding strategy when n is unknown but k is known requires bidders to update their bid function as the price moves down. As bidders observe items being claimed, they gain information about the group size and thus update their priors P about that size. Let t be the number of clock ticks since the start of the auction. The number of units remaining k_t and the current clock price c_t are signals that inform bidders about their competitors.

Subjects begin the auction with common priors P about their group size. For example, at the start of each auction in our experiment, the group sizes 4 and 8 are equally likely. Intuitively, as the price descends with each clock tick (without any units being claimed) it becomes more likely that the group is smaller because larger groups would likely have higher maximum values resulting in units being claimed sooner. If no units were claimed, the probability that the group is actually equal to the size of the smallest n in N would increase and eventually asymptote to 1. When a unit is claimed, there is a jump in the iterative updating of agents' beliefs: the probability of the larger group adjusts upward. These jumps are especially large when units are claimed with high bids. A complete proof of the belief updating procedure is provided in Appendix B.

Since bidding strategies are symmetric, buyers assume that their opponents are also updating their beliefs. Therefore the optimal bid function $B(v)$ changes as a function of beliefs about group size, which in turn are a function of the current clock price c_t, the current number of remaining units k_t, and their updated priors P_{t-1}. The current clock price c_t is a meaningful signal even if no items have yet been claimed, because participants know that items have not been claimed (in contrast to the unknown k treatment where the clock could stop at any moment). Bidding decisions are made conditional on the observed signals and the most recently updated beliefs about group size P_{t-1}.

With updating, the bidding strategy becomes a function of the discrete information stream:

$$B_t(v, N, P_{t-1}, c_t, k_t) = v \frac{\sum_{g=1}^{G} \frac{n_g - m}{n_g - m + 1} p_{g_{t-1}} v_i^{n_g} \binom{n_g - 1}{m - 1}}{\sum_{g=1}^{G} p_{g_{t-1}} v_i^{n_g} \binom{n_g - 1}{m - 1}} \tag{3}$$

The strategy with updating is not the same as the strategy when n and k are unknown. However, the bidding limit would never be located above the bid/value ratio suggested by the largest group size n in N or below the ratio suggested by the smallest group size. Thus, the bidding limit would stay in the cone charted in Figure 3. Thinking of P as a function of the data stream provided by c_t, the function has discontinuities when k_t changes. For example, if $N = \{3, 6\}$ and $P = \{0.5, 0.5\}$ then agents begin the auction believing that there is a 50% probability that they are in a group of 6. Based on order statistics, there is a predicted price at which the agent with the highest value in a group of 6 would place a bid. If the price drops below this amount and no items have been claimed, then agents would gradually adjust their beliefs so that it is more likely that they are in a group of 3. However, if an item were claimed at that price, then agents would suddenly shift their beliefs to reflect the higher probability that they are in a group of 6.

Given the predictions from the equilibrium strategies given in equations (1) - (3), we next design an experiment to test the potential difference in these information conditions.

3 Experimental Design

We conducted 16 experimental sessions using a total of 128 subjects. In each session, 8 subjects participated in 16 Dutch auctions, which we call "rounds". Hereafter we will use the terms "round" and "auction" interchangeably. Before each round, subjects were randomly sorted into groups of 4 or 8. The two group sizes were equally likely to be selected in a round. In each auction there were 3 units available and subjects had a positive value for a single unit. Subjects were assigned private values drawn randomly from a discrete uniform distribution over [1, 100]. The exact values used in our experiment can be found in Appendix C (Table C.1).

The auction began at a price of 100 tokens and decreased by 2 tokens every 2 seconds.[6] As soon as the third unit was claimed, the auction stopped and the three winners each received a unit at the current price. The profit of each subject was her value minus the uniform price at which the auction stopped. A screenshot of the Group/Units auction interface can be found in Appendix C (Figure C.1).

Subjects could place a bid in either of two ways:

1. Instant Bid - a subject could immediately accept the current price.

2. Proxy Bid - a subject could privately enter a bid below the current price and wait for the price to cross her bid. Subjects were allowed to change their proxy at any time as long as they had not claimed an item.

The treatments only differed in the information provided to bidders during the auction. Our four treatments were:

Group/Units	Groups size is shown and number of units remaining is shown.
No Group/Units	Groups size is not shown and number of units remaining is shown.
Group/No Units	Groups size is shown and number of units remaining is not shown.
No Group/No Units	Groups size is not shown and number of units remaining is not shown.

Prior to entering the auctions, participants learned about the auction mechanism through instructions (documented in Appendix D), a quiz that would not allow them to proceed until they entered correct answers, and one practice round that did not count toward their profits. Subjects were paid in cash at the end of the 16 auctions. Not including a $7 payment for showing up on time, earnings for the 40-minute experiment ranged from $6 to $14 with a mean of $9.15.

[6] In our experiment, 24 tokens = 1$

	Group	*No Group*	Marginal	Effect	p-value
Units	41.21	41.24	41.23	*Units*	0.993
	(1.15)	(.30)		*Group*	0.017
No Units	38.24	44.23	41.24	*Interaction*	0.018
	(1.70)	(.62)			
Marginal	39.73	42.74	41.23		

Table 1: **Average Observed Prices by Treatment (st. errors) and 2x2 ANOVA Results (df = 15)**

4 Results

4.1 Auction Prices and Efficiency

Each auction round resulted in a uniform price determined by the 3^{rd}-highest bid. Prices are higher in the groups of size 8 because the marginal or 3^{rd}-highest value in a group of 8 is typically larger than the 3^{rd}-highest value in a group of 4. We begin the analysis by averaging the prices across all rounds within a treatment so that each treatment is an independent unit of observation. Table 1 shows the average price at which the clock stopped in each treatment and the effects of showing group size and showing the remaining units.

Finding 1. Showing group size lowers revenue.
Evidence: In Table 1, showing group size lowers prices by 7% on average (p-value = 0.017). Note that the effect is entirely driven by the difference between prices in the *No Units* treatment.

Finding 2. Revenue generated is largest when both group size and units remaining are not revealed.
Evidence: The p-value on the interaction term in Table 1 is significant, so there is not a simple linear additive effect for the two treatment variables. This stems from the significant overall effect of showing group size even though size information has no effect when the number of units remaining is shown.

We find that the different institutions significantly affect bids. Using the F-test we can reject the null hypothesis that all four of the cell means are equal. $F(15, 3) = 6.343, p = 0.005$. In Table 1 it appears that No Group/No Units has consistently higher prices. We confirm this observation using the linear mixed effects model in equation (4) to examine the effect of treatment on observed auction prices. The treatment effects are modeled as (zero-one) fixed effects. For example, $GroupUnits_i$ takes the value of 1 if the observation is in the treatment where group size and units remaining are both shown, and 0 otherwise. The 16 independent sessions are modeled as random effects, u_i. We estimate the model

$$P_{ij} = \alpha + u_i + \beta_1 GroupUnits_i + \beta_2 NoGroupUnits_i + \beta_3 GroupNoUnits_i + \varepsilon_{ij} \qquad (4)$$

where P_{ij} is the uniform price from the j^{th} auction in the i^{th} session. The mean of the treatment with no information, *No Group/No Units*, is the intercept α. The beta coefficients provide the difference from the *No Group/No Units* baseline mean, or the effect of revealing information. The estimates from these regressions are reported in Table 2. We also report regressions on price for the subset of observations for the two distinct group sizes labeled "$n = 4$ Prices" and "$n = 8$ Prices" in Table 2. We show the results for small and large groups separately because the theoretical prediction for the effect on prices differs depending on group size. Recall that there are twice as many observations of groups of size 4 because the subjects are switched between auctions with 8 bidders and two simultaneous auctions with 4 bidders each.

Finding 3. Relative to providing no information, treatments where information is shown have lower revenue with the largest effect in the *Group/No Units* treatment.
Evidence: The treatment coefficients in Table 2 are negative. This indicates that prices in treatments where information is provided are generally lower than those in *No Group/No Units*. When we test for treatment effects separately by group size, the negative coefficients tend to be the most significant for auctions where group sizes were small due to higher variance of outcomes between the groups of 8. The negative effect of *Group/ No Units* is strongly significant for both group sizes.

The most efficient outcome occurs when the subjects with the three highest values in a given auction win the three available items. If subjects with lower values can "sneak" into the top three bids, there will be a loss of efficiency. Our next finding examines allocations in our auctions and where misallocations occur.

Finding 4. Providing subjects with information lowers efficiency.
Evidence: In Table 3 we analyze the average efficiency across all auctions in each of the 16 sessions and we see a similar pattern to that of the ANOVA for average prices. Auctions with the highest prices have the least efficiency loss. The effect of showing group size is statistically significant, and it is driven primarily by the difference between Group/No Units

	All Prices	All Prices	n=4 Prices	n=8 Prices
α = No Group/No Units	38.83	35.11	38.53	55.63
	(1.12)	(2.16)	(.994)	(1.66)
β_1 = Group/Units	-3.02**	-3.02**	-3.22**	-2.63
	(1.53)	(1.53)	(1.43)	(2.34)
β_2 = No Group/Units	-3.00**	-3.00**	-2.81**	-3.34
	(1.53)	(1.53)	(1.43)	(2.34)
β_3 = Group/No Units	-6.00***	-6.00***	-4.80***	-8.38***
	(1.53)	(1.53)	(1.43)	(2.34)
Round	-	0.70	-	-
		(0.43)		
Round Squared	-	-0.03	-	-
		(0.02)		
Group Size Dummy	16.48***	16.48***	-	-
	(0.87)	(0.87)		
No. Observations	384	384	256	128
Number of Groups	16	16	16	16
R-squared	0.50	0.50	0.05	0.10

*10% significance level, **5% significance level, ***1% significance level
Standard errors in parentheses.
Errors are clustered by session.

Table 2: **Treatment Effects on Auction Prices**

	Group	*No Group*	Marginal	Effect	p-value
Units	0.97	0.97	0.97	*Units*	0.82
No Units	0.96	0.99	0.97	*Group*	0.05
Marginal	0.97	0.98	0.97	*Interaction*	0.05

Table 3: **Average Efficiency by Treatment and 2x2 ANOVA Results (df = 15)**

and No Group/No Units.

In theory, if everyone is bidding symmetrically, this mechanism should be fully efficient. However, misallocations can occur if someone with a high value did not bid higher than someone else, for instance if the 4^{th} highest value subject claimed the last item. Figure 4 shows how many misallocations occurred in each treatment for each group size by providing the count of the misallocations when the highest, second highest and marginal value bidders are displaced. For example, the two bars to the left in Figure 4 show the misallocations for the treatment where the group size ($n = 4, n = 8$) is known as well as units remaining. In this case, for group size 4, the marginal bidder was displaced in 7 of the 64 auctions (16 auctions per session and 4 sessions per treatment). For group size 8 in that treatment, lower-valued bidders displace higher-valued bidders 8 times during the 32 auctions (recall that there are twice as many $n = 4$ auctions because two are played simultaneously).

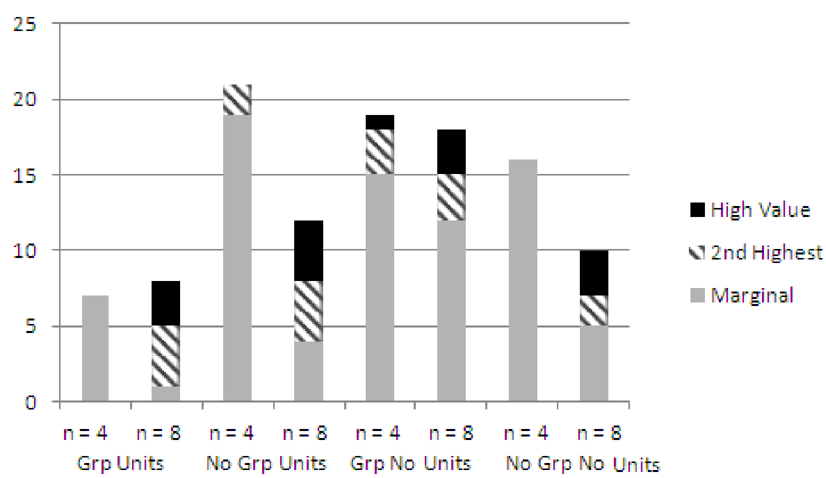

Figure 4: **Number of Misallocations**

It was most common for the person with the 3rd highest value, who we call the "marginal" bidder, to fail to secure an item. We also observe that subjects with the highest and 2^{nd} highest values get cut out of the auction more often in group sizes of 8. If the marginal bidder has a value that is close to the 4^{th} highest bidder, their loss will have a small impact on efficiency. The efficiency loss from a high valued bidder not winning an item, especially in groups of 4, can be very large. It is rare for the subject with the highest value not to be allocated an item in a group of 4, but we did observe it once in the *Group/No Units* treatment, which resulted in a significant efficiency loss.

All treatments produce highly efficient outcomes. In the next sections we show that subjects are bidding quite high, almost to their values, relative to the Nash predictions.

4.2 Observations Compared to Theoretical Predictions

Figure 5 presents the deviations in observed prices from the theoretical predictions for each round with prices averaged over the observations in the four sessions, separated by treatment. Exact theoretical predictions for each round, based on the value assignments, can be found in Appendix C. The horizontal axes in Figure 6 do not progress by even intervals because subjects were not switched between group sizes in a predictable pattern. See Appendix C for the order of the group assignments and the value parameters.

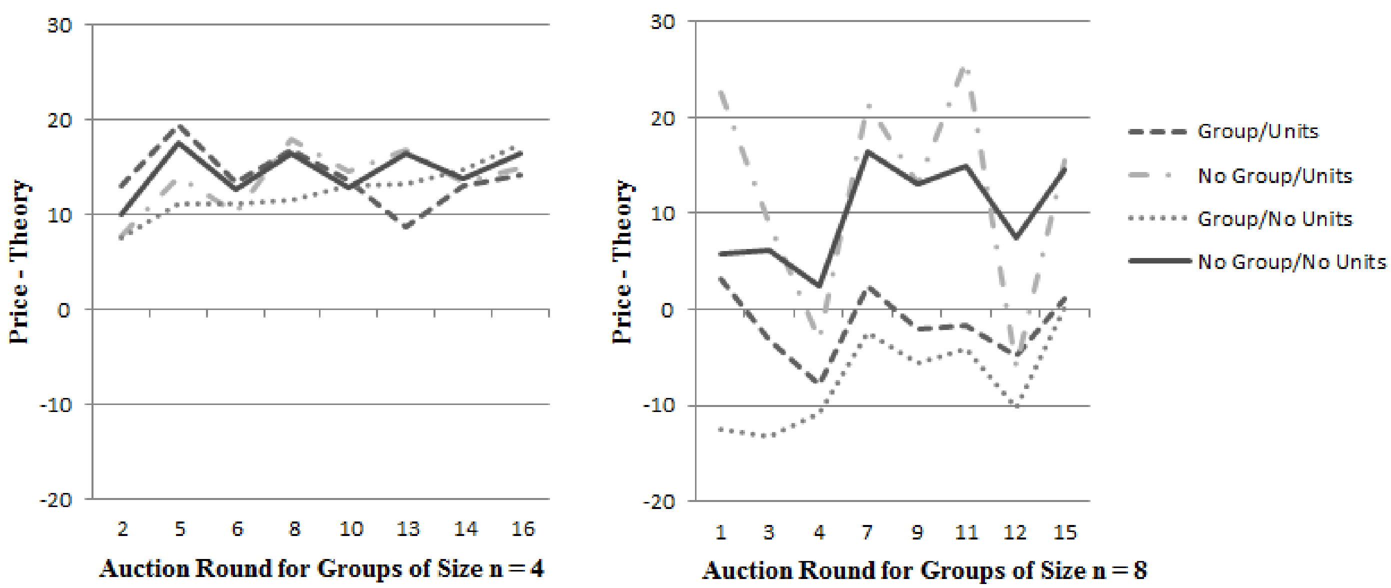

Figure 5: **Differences Between Average Final Bids and Theoretical Predictions**

We can reject the null hypothesis that each cell mean equals the theoretical point prediction, F-stat (4,12) = 97.53, p-val=0.000. The cell means do vary significantly from one another, but they are not well explained by the theory. When we use t-tests to compare prices to point predictions for individual treatment cells, we reject every null hypothesis of equality except for the case of group size 8 in *Group/Units* (p-val=0.48, two-sided). Although the observed prices were higher than the point predictions, the relevant directional predictions of the theory were both confirmed as we show in Findings 5 and 6.

Finding 5. Showing units lowers prices when group size is not known.
Evidence: We can reject the null hypothesis that price, when units are shown, is greater than or equal to price when units are not shown, in the case when group size is unknown

(p-val = .0053, one-sided). This is true for both groups pooled and for each group size individually.

Finding 6. Showing units has no effect on price when group size is known.
Evidence: We cannot reject the null that price, when units are shown, is equal to price when units are not shown, in the case when group size is known (p-val = .2056, two-sided). This is true for all group compositions.

We observe that bids are overwhelmingly above the predicted price when group size is 4 where the equilibrium strategy is for subjects to bid half of their value. This same overbidding relative to the prediction occurs when $n = 8$ in the *No Group/No Units* and the *No Group/Units* treatments. This is because the theory indicates that these subjects, who are uncertain of their group size, should shade their bids to capture the possible profit from being in a group of 4. However, if the subjects do not lower their bids when they know they are in a group of 4, it is consistent that they would not lower their bids when it would be beneficial for them to assume they are in a group of 4.

Although there exists an equilibrium strategy, groups fail to reach it. In our case, the equilibrium outcome is much like auction collusion where each person bids as low as they can without incurring too much risk of losing the item. However, if one person deviates from the strategy, that person will be able to snipe units from subjects who do wait as the price goes down. This is consistent with the findings of Brown et al. (2009) who created a "collusion incubator" that fosters tacit collusion in an ascending-price institution. When they switched the auction to a descending-price institution mid-experiment, prices returned to competitive levels even though participants had just been successfully colluding. They conclude that the Dutch auction is a collusion destroyer.

We next examine further dimensions of bidding behavior in these auctions.

4.3 Bidding Behavior

Subjects can bid in two ways: Instant Bid and Proxy Bid. By Instant Bidding, they immediately accept the current clock price. If there is only one unit left, an Instant Bid will stop the auction.

Finding 7. More participants stop the clock with an Instant Bid when units remaining are shown.

	Group	*No Group*	Marginal	Effect	p-value
Units	19.0 (1.58)	12.0 (3.56)	15.5	*Units* *Group*	0.05 0.149
No Units	10.3 (3.30)	8.8 (2.17)	9.6		
Marginal	14.7	10.4	12.5		

Table 4: **Average Instances of Stopping the Clock with Instant Bids (stand. error) and 2x2 ANOVA Results (df = 15)**

Evidence: See Table 4. The effect of showing units is an average increase of 6 more Instant Bids per session (p-value = .05).

This can be taken as evidence that subjects are making their decision based on the new information provided by seeing units being claimed (i.e., jumping in when only one unit is left).

Next we ask whether experience changes outcomes over the course of the 16 auction rounds. To investigate whether prices trend up or down, we add to the original linear mixed effects model a term for the round index that ranges from 1 to 16. We include the squared value of the round number to allow for nonlinear trend effects. When we estimate

$$\begin{aligned} P_{ij} = \alpha + u_i + \beta_1 GroupUnits_i + \beta_2 NoGroupUnits_i + \beta_3 GroupNoUnits_i \\ + \beta_4 Round_{ij} + \beta_5 Round_{ij}^2 + \varepsilon_{ij} \end{aligned} \tag{5}$$

where P_{ij} is the price from the j^{th} auction in the i^{th} session, the effect of *Round* is not significant. The estimates are shown in the last column of Table 2. We conclude that the trend in prices over time is not important in explaining our data.

5 Conclusion

A set of multi-unit Dutch auction experiments was conducted to test the effect of providing two items of information to bidders - the number of bidders in the auction and the number of units remaining out of a fixed supply. Overall, we find that providing information lowers prices. In particular, in the most relevant case in which the number of bidders is unknown, the largest negative effect on both revenue and efficiency is when the number of units remaining is provided to bidders. While we find that Nash equilibrium predictions are consistent with the directional changes in our treatments, subjects consistently overbid relative to theory and about 20% of bids were equal to the subject's value. We also find evidence that subjects are using information about the number of units remaining to update their bidding strategies since there is a marked increase in the use of instant bidding over proxy bidding in the treatments where units remaining is revealed.

Our results provide a recommendation for designing a multi-unit Dutch auction that will maximize revenue and efficiency. Providing less information generates higher prices and results in higher efficiency because there are fewer misallocations of items to bidders with low values. More importantly, we note that in most real-world auctions the real number of bidders participating in an auction for a specific item is unknown. We have taken a first step in extending the theory of a Dutch auction with this important feature being present. We have also examined how information interacts in this case in terms of the auction design. Our experiment constitutes the first systematic attempt to understand these interactions that relate directly to what is encountered by those creating such auctions in the field.

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Appendix A

Proof of equation (2): Bidding Strategy If n Is Unknown and k Is Unknown

The derivation of the risk neutral symmetric Nash equilibrium (RNSNE) bidding strategy in a multi-unit auction generalizes the derivation of the bidding strategy in a single-unit first-price sealed bid auction. To calculate the probability of winning an item in a multi-unit auction, we first determine order statistics for agents' values.

Let $X_1, \ldots, X_n$ be independent random variables, each having CDF $F(x)$. Let $F(k)(x)$ $(k = 1, \ldots n)$ denote the CDF of the k^{th} order statistic $X^{(k)}$. The probability that at least r of the other draws from a distribution $F(x)$ are less than or equal to a number x is

$$P[X^{(k)} \leq x] = \sum_{s=0}^{k-1} \binom{n-1}{s} F(x)^{n-1-s}[1 - F(x)]^s.$$

The agents want to find the probability that their bid b_i will be the winning bid. For a symmetric strategy, that is equivalent to determining whether they have a higher value than at least k of their competitors. The probability that at least k of the other values $v_j = B^{-1}(b_j)$ drawn from a distribution with CDF $F(x)$ are less than or equal to v_i, where $i \neq j$ is

$$P[B^{-1}(b_j)^{(k)} \leq v_i] = \sum_{s=0}^{k-1} \binom{n-1}{s} F(B^{-1}(b_i))^{n-1-s}[1 - F(B^{-1}(b_i))]^s.$$

Let $Z(b) = F(B^{-1}(b))$. Note that $B^{-1}(b)$ maps a bid into the value v that would generate that bid, and $F(v)$ is the probability that a randomly drawn value is less than v. The function $Z(b)$ is undefined for values of b that are greater than the highest strategic bid. The probability that a bid will win against the k^{th} highest opponent's bid is

$$\psi_k(b, n) = \sum_{s=0}^{k-1} \binom{n-1}{s} Z(b)^{n-1-s}[1 - Z(b)]^s,$$

where $0 \leq \psi_k(b, n) \leq 1$.

Note that if $k = 1$, this reduces to the simple CDF of the first-highest order statistic $Pr[b_j \leq b_i \ \forall \ i \neq j] = Z(b_i)^{n-1}$.

When n is unknown, we extend the model to account for the possible group sizes $N =$

$\{n_1, \ldots n_G\}$ and their corresponding probabilities $P = \{p_1, \ldots p_G\}$:

$$\begin{aligned}\Phi_k(b_i, N, P) &= \sum_{g=1}^{G}[p_g \cdot \sum_{s=0}^{k-1} \binom{n_g - 1}{s} Z(b_i)^{n_g-1-s}(1 - Z(b_i))^s] \\ &= \sum_{g=1}^{G}[p_g \cdot \psi_k(b_i, n_g)]\end{aligned}$$

where $0 \leq \psi_k(b_i, n_g) \leq 1$ and $\sum_{g=1}^{G} p_g = 1$.

At this point we've introduced the symmetric bid strategy property, so that in the remainder of the proof, the agent's own bid is the only bid considered. We can therefore suppress the subscript i for the remainder of the proof.

Note: The ψ_k function would be greater than one if b were greater than the maximum bid predicted for a given n_g. This would happen for the smaller groups. So we truncate ψ_k at one.

We can now compute the agent's expected utility in an m-unit auction

$$Eu(b, N, P) = (v - b)[\Phi_m(b, N, P) - \Phi_{m-1}(b, N, P)] + \int_0^b (v - \omega)\frac{d}{d\omega}\Phi_{m-1}(b, N, P)d\omega. \quad \text{(A.1)}$$

Expected utility is zero if the buyer does not win the item. A buyer can win in the auction either as the price setter with the last accepted bid or as one of the higher bidders. The first term on the right side of equation (A.1) is the case where the buyer is the price setter. In that case the buyer earns $(v - b)$. The factor that multiplies $(v - b)$ is the probability that the buyer is the price setter. If the buyer wins an item with a bid above the stopping price, her profit is her value minus the uniform price. Her profit in that case is the integrand in the integral from the second term in equation (A.1). The probability that a buyer's bid is greater than or equal to the winning bid is $\Phi_{m-1}(\omega, N, P)$.

To maximize expected utility, we solve $\frac{d}{db}Eu(b, N, P) = 0$. Differentiate equation (A.1), noting that the derivative of the integrand on the right is $(v - b)\Phi_{m-1}(\omega, N, P)$ by the fundamental theorem of calculus. Cancel terms to obtain the equation

$$\frac{d}{db}Eu(b, N, P) = (v - b)\Phi'_m(b, N, P) - \Phi_m(b, N, P) + \Phi_{m-1}(b, N, P). \quad \text{(A.2)}$$

Set the derivative in equation (A.2) equal to zero and rearrange terms to get

$$(v - b)\Phi'_m(b, N, P) = \Phi_m(b, N, P) - \Phi_{m-1}(b, N, P). \quad \text{(A.3)}$$

As shown in Appendix D, $\Phi'_m(b) = (n-m)(\Phi_m(b) - \Phi_{m-1}(b))\, Z'(b)/Z(b)$. If we make this substitution into equation (A.3) then

$$(v-b)\sum_{g=1}^{G}[p_g(n_g-m)(\psi_m(b,n_g)-\psi_{m-1}(b,n_g))\, Z'(b)/Z(b)] = \Phi_m(b,N,P)-\Phi_(m-1)(b,N,P). \tag{A.4}$$

Assume $F(v) = v$ so that $Z(b) = F(B^{-1}(b)) = B^{-1}(b)$. Then $Z(b) = v$. Since $Z(b) = B^{-1}(b)$ under the assumption that $F(v) = v$ we get $Z'(b) = (B^{-1})'(b)$. Differentiate the identity $B(B^{-1}(b)) = b$ to get $B'(B^{-1}(b))(B^{-1})'(b) = 1$. This can be written as $B'(v)(B^{-1})'(b) = 1$ so $(B^{-1})'(b) = 1/B'(v)$. Therefore $Z'(b)/Z(b) = 1/(B'(v)\cdot v)$.

Substitute $b = B(v)$ into equation (A.4) and use the fact that $Z'(b)/Z(b) = 1/(B'(v)\cdot v)$ to get

$$(v-B(v))\sum_{g=1}^{G}[p_g(n_g-m)(\psi_m(B(v),n_g)-\psi_{m-1}(B(v),n_g))\frac{1}{B'(v)\cdot v} = \Phi_m(B(v),N,P)-\Phi_{m-1}(B(v),N,P)$$

. We find the optimal bidding strategy by solving the following differential equation

$$\frac{v-B(v)}{B'(v)\cdot v} = \frac{(\Phi_m(B(v),N,P)-\Phi_{m-1}(B(v),N,P))}{\sum_(g=1)^G[p_g(n_g-m)(\psi_m(B(v),n_g)-\psi_{m-1}(B(v),n_g)]}$$

.

The solution $B(v,N,P)$ to this differential equation is the RNSNE strategy

$$B(v,N,P) = v_i\frac{\sum_{g=1}^{G}\frac{n_g-m}{n_g-m+1}p_g v^{n_g}\binom{n_g-1}{m-1}}{\sum_{g=1}^{G}p_g\, v^{n_g}\binom{n_g-1}{m-1}}$$

.

Appendix B

Proof of equation (3): Bidding Strategy If n Is Unknown and k Is Known

Agents update their strategies using information from the auction to improve their estimates of the size of the group they are competing against. Agents revise P based on the observed information brought by each successive k_t they observe at clock time c_t. Then they make their bids by plugging P into the RNSNE strategy for unknown n.

To construct the risk neutral symmetric Bayes-Nash equilibrium (RNSBNE) bidding strategy for the case when k is known, it is necessary to first define the process by which agents update P, the vector of probabilities that the group size is a particular n out of N.

Agents use Bayes' rule to determine the probability of the event E_i that the group size is $n = n_h$ conditional on the observed data D_t that when the clock reaches c_t the number of units claimed is k_t

$$\begin{aligned} Pr(E_i|D_t) &= \frac{(Pr(D_t|E_i)Pr(E_i))}{Pr(D_t)} \\ &= \frac{(Pr(D_t|E_i)Pr(E_i)}{\sum_j Pr(D_t|E_j)Pr(E_j)}. \end{aligned}$$

The current data $D_t = \{c_t, k_t\}$ tell the bidders more about their environment than they knew before the auction started. The probability that n is a certain $n_h \in N$ given $\{c_t, k_t\}$ is

$$Pr(n = n_h|\{c_t, k_t\}) = \frac{Pr(c_t, k_t|n = n_h) \cdot (p_h)_{t-1}}{\sum_{g=1}^{G} Pr(\{c_t, k_t\}|n = n_g)(p_g)_{t-1}}.$$

where the prior probability (at time $t-1$) that the group size is n_h is $Pr[n = n_h] = (p_h)_{t-1}$.

When agents observe bids that are generated by the common RNSBNE strategy, they can use the inverse bid function to determine the value that generated the bid. Observed bids could be generated by any group size, but larger groups are more likely to have bidders with high values who will place higher bids. Until the first unit is claimed, while $k_t = 0$, every bidder is updating P to gradually favor the smallest possible group size.

To use Bayes' rule, bidders need to find the probability of the observed data if n were n_h. Recall that the probability that a bid will win against the k^{th} highest opponent's bid is

$$\Psi_k(b, n) = \sum_{s=0}^{k-1} \binom{n-1}{s} Z(b)^{n-1-s}[1 - Z(b)]^s$$

where $0 \le \Psi_k(b, n) \le 1$. This is the probability that no more than $k-1$ bids have been placed.

The probability that no more than k bids have been placed yet is

$$\Omega_k(c_t, n) = \sum_{s=0}^{k} \binom{n-1}{s} Z(c_t)^{n-1-s}[1 - Z(c_t)]^s$$

which is similar to Ψ except that the summand iterates one more time.

Therefore the probability that exactly k bids have been placed is

$$Pr(\{c_t, k_t\}|n = n_h) = \Omega_k(c_t, n_h) - \Omega_{k-1}(c_t, n_h)$$

This simplifies to

$$Pr(\{c_t, k_t\}|n = n_h) = \binom{n_h - 1}{k} Z(c_t)^{n_h - 1 - k_t}[1 - Z(c_t)]^k$$

The updated probability $(p_h)_t$ of group size n_h is defined as

$$\begin{aligned}(p_h)_t &= \frac{Pr(\{c_t, k_t\}|n = n_h) \cdot (p_h)_{t-1}}{\sum_{g=1}^{G}[Pr(\{c_t, k_t\}|n = n_g) \cdot (p_g)_{t-1}} \\ &= \frac{\binom{n_h-1}{k} Z_t(c_t)^{n_h-1-k}[1 - Z_t(c_t)]^k (p_h)_{t-1}}{\sum_{g=1}^{G}[\binom{n_h-1}{k} Z_t(c_t)^{n_g-1-k}[1 - Z_t(c_t)]^k (p_g)_{t-1}]} \end{aligned} \tag{B.1}$$

This function is undefined for clock prices above the highest rational bid. No updating should happen until the clock has ticked past that bid, and all probabilities should remain at the initial prior probabilities until the clock crosses that threshold.

We use the elements above to construct the bid function

$$B_t(v, N, P_{t-1}, c_t, k_t) = v \frac{\sum_{g=1}^{G} \frac{(n_g-m}{n_g-m+1} p_{g_{t-1}} v^{n_g} \binom{n_g-1}{m-1}}{\sum_{g=1}^{G} p_{g_{t-1}} v^{n_g} \binom{n_g-1}{m-1}} \tag{B.2}$$

When updating, two things change. The P must be referenced from the last round and the $Z(c_t)$ changes because it is a function of the new P. Because this strategy is symmetric, one's opponents are also updating their beliefs and strategies, so it can be assumed that all agents share the same updated P.

To summarize, the RNSBNE strategy involves the following iterative process:

1. At time t, use P_{t-1} to make a bidding decision, namely whether to accept the current clock price or keep waiting.

2. Observe whether k changes.

3. Use the new data $\{c_t, k_t\}$ to determine a new P. Assume competitors have done the same.

4. Repeat steps 1 through 3 with each tick of the clock.

Appendix C

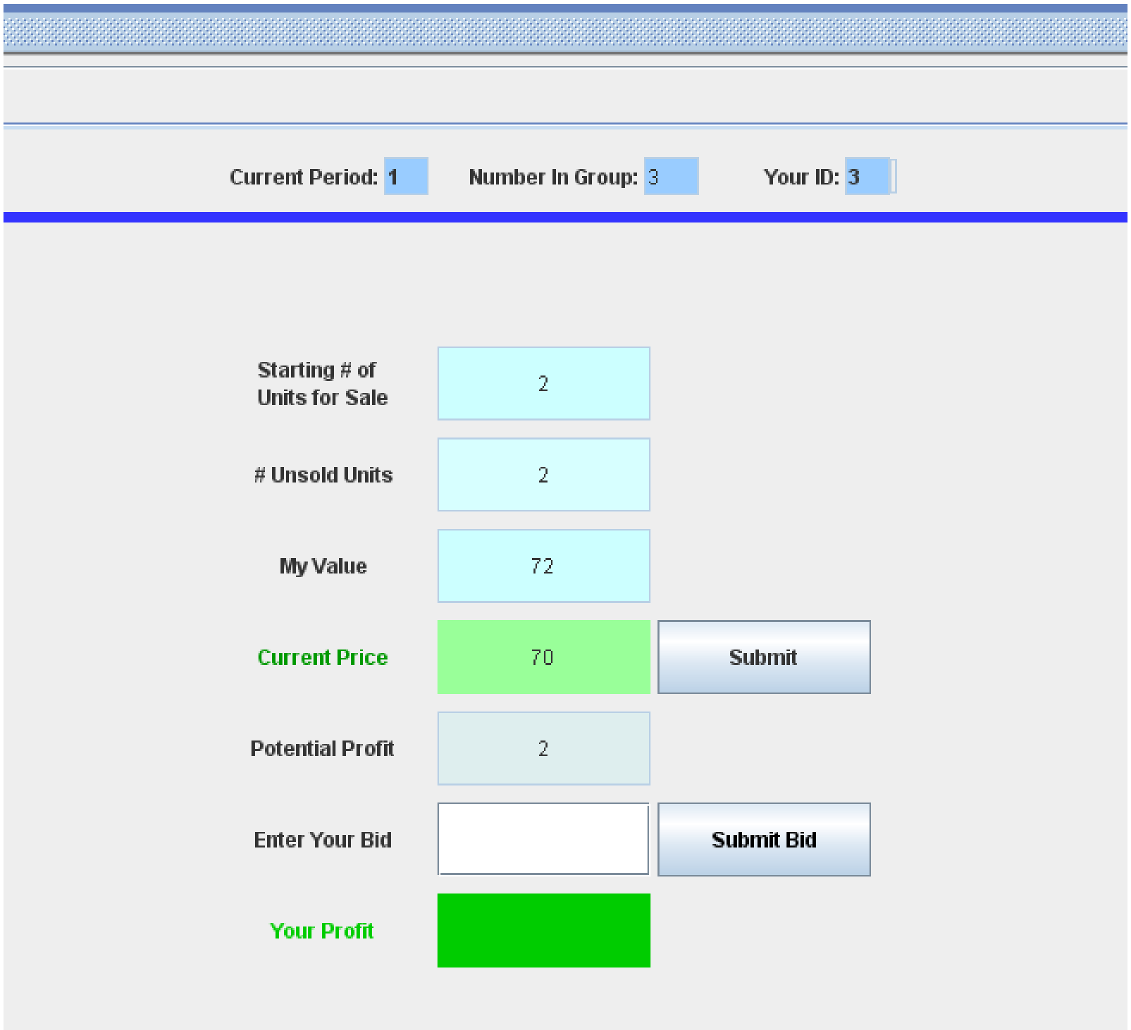

Figure C.1: **Auction Subject Interface**

The sequence in Table C.1 of 8 sets of values and group sizes is repeated twice for 16 auctions total. Subjects are shuffled so that they do not experience the same value assignment in the same group twice.

Experimental Parameters			Predicted Prices from Theory		
			Known n	Unknown n	
Round	n	Values		Known k	Unknown k
1	8	10, 12, 20, 36, 56, 66, 84, 88	55	36	46
2	4	28, 42, 66, 80	20	22	24
3	8	20, 26, 46, 46, 54, 68, 76, 90	56	36	48
4	8	16, 18, 36, 48, 60, 76, 84, 94	64	62	56
5	4	12, 46, 74, 88	24	24	26
6	4	32, 34, 56, 56	16	18	18
7	8	12, 22, 28, 38, 50, 52, 60, 62	44	26	32
8	4	26, 48, 60, 90	24	24	28

Table C.1: **Assigned values and theoretically predicted prices for each auction round**

Appendix D Experiment Instructions

These instructions were presented to subjects in a series of PowerPoint slides:

- Introduction
- This is an experiment in market decision-making. If you follow the instructions carefully and make good decisions, you can earn a considerable amount of CASH.
- This experiment consists of many auctions. At the end you will receive your total earnings from all auctions in addition to your $7 show-up fee.
- Do not talk or communicate in any way with the other people during the experiment.
- Auction Description
- There are 8 participants in the experiment with you. You will be randomly sorted into a group with either of 4 or 8 participants for each auction.
- There are 8 participants in the experiment with you. You will be randomly sorted into a group with either of 4 or 8 participants for each auction.
- Your earnings in an auction, which are yours to keep, is 0 if you do not win an item. If you receive an item, your profit is: (your value) - (price paid)
- Value of the Items
- Every person in each auction will have a value between 1 and 100. Each number from 1 to 100 has an equal chance of being someone's assigned value.
- Imagine that there are 100 marbles in a black jar labeled 1 through 100.
- For each auction, it is as if you blindly put your hand in the jar and pick out a marble and the number becomes your value for that round.
- So, your chance of drawing a value between 31 and 40 is 10%.
- Once you have picked out a marble it is replaced in the jar and someone else goes through the same selection process for their value.
- Price You Pay
- In this auction all three people who win an item pay the same price.

- The process begins at a listed price of 100 experimental dollars (E$).
- The price then decreases by 2E$ every 2 seconds.
- A participant can BID for a unit by either:
    1. Saying yes to the current price, or
    2. Type in an amount they would say yes to if the price were to reach that amount
- Example
- When 3 people have a BID equal to or higher than the current price, the process will stop and those three people will each receive a unit at the current clock price.
- If the price is 65 and two participants have already said yes to higher prices and a third person BIDS 65, then
- The auction will end and those three people who are "in" will each receive one item for a price of E$65. Their profit will be: (their value) - 65
- How To Bid
- Enter a number in the bid field that is equal to or lower than the current clock price and click [Submit Bid].
- Or you can put in a bid at exactly the current clock price by clicking [Submit].
- You may change your bid only if it is lower than the current clock price. Do this by highlighting your number, typing a new number, and clicking [Submit Bid].
- You will earn profits in experimental dollars during the auctions. At the end of the experiment, your will receive one $US for every 24 E$.
- The FIRST auction is a PRACTICE ROUND. The practice round does not count toward your earnings, all others do.
- Please raise your hand if you have a question at any time and someone will come to assist you.