Contracting in Anticommon Settings: Theory and Experiments

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Abstract:

"Hold-up" problems may occur in any sequential economic exchange that requires agreement by multiple agents with individual incentives. When a buyer purchases complementary properties from different sellers, hold-ups occur because later sellers gain bargaining power. This issue arises frequently with patents, land redevelopment, and other "anticommon" settings. This paper examines the predicted behavior and welfare of agents under several contingent purchase bargaining rules including a "sealed simultaneous bid" rule and a "sequential contingent package offer". Independent sellers should have incentive to collectively agree to contingent bargaining as it improves their expected profits despite the perceived gains from being a later seller. These findings are then tested experimentally and results support the theoretical finding that agents capture the most surplus under the "sealed simultaneous bid" rule.

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JEL: [C72], [C78], [C91], [C92], [D82]

1. Introduction

Michael Heller coined the term “tragedy of the anticommons” to describe an environment where socially desirable outcomes may be unattainable because of necessary bargaining between numerous rights holders with individual incentives [7]. The anticommons concept provides a unifying framework for multiple agent contracting failures that have been observed in land redevelopment, patents, and other markets where there is a single buyer interested in purchasing two or more unique and complementary properties from individual sellers.

Hold ups may occur if the buyer must make separate bilateral transactions with each of the independent sellers. For example, consider a land developer who has identified a tract of land for construction and needs to purchase the entire set of properties in order to build at that location. If the developer finds that one of the home owners is offering an “unreasonably” high amount for this home, the developer may make smaller bids for the other homes due to fear of being “held-up”. Hold-ups occur if “first movers” are unwilling to “commit” to (or under invest in) a transaction. The reason for this reluctance is that if the first mover commits, then some “last movers” gain disproportionate bargaining power. This added power reflects the fact that if a buyer purchases all the properties with the exception of one that is owned by a stubborn home owner – this owner would have more leverage in bargaining.

After a buyer purchases properties, the cost of these purchases is sunk when deciding whether to purchase additional ones. When deciding to purchase additional property, she[1] is willing to pay a cost up to the additional value for acquiring the complementary property. When the properties are perfect complements, this additional value for that final missing property is the complete package value. Before initiating any property acquisition, the buyer recognizes that she could later end up in a position where she is willing to pay for her total possible gain and consequently take a loss on the project overall. To compensate for the possibility of being “held-up”, the buyer will only make lower offers to first sellers[2]. Hold-ups decrease efficiency because the initially low buyer offers may prevent mutually beneficial contracting from occurring at all.

In addition to welfare loss from the potential of hold-up, a setting in which multiple agents have monopoly power also makes it difficult to contract in this environment. Myerson and Satterthwaite (1983) find that no two person incentive compatible and individually rational bargaining game can satisfy ex-post efficiency [10]. Thus, there is some inherent efficiency loss

[1] The buyer will be referred to as “she” and each seller as “he”

[2] Holmstrom and Roberts describe this general hold-up finding as the "most influential work" in recent decades on why firms exist and what determines their boundaries [8]

under bargaining with just two non-cooperative agents even in static (without hold-ups) environments.

The anticommon setting is more complex because there is a single buyer and *multiple* sellers. If the properties are unique and completely complementary and there exists some type of contingent purchase contract, then all agents effectively have monopoly bargaining power with each possessing the leverage to prevent a deal. Although this contingent contract prevents a hold-up from occurring, it creates this new multiple agent bargaining problem. Under these assumptions, the amount that the buyer pays now depends on the offers made by all sellers. The amount that each seller receives depends on the buyer's offer and the offers made by all other sellers.

If the regulator's only concern is to maximize efficiency, then the regulator could decide to confiscate all property under eminent domain for land redevelopment or force compulsory licensing for patents. These actions, however, would violate the seller's participation constraint and create new potential issues. For example, severely limiting the patent system would dramatically change the incentives for innovation and frequently enforcing eminent domain could significantly change home values. Either extreme action could easily be viewed as a restriction of fundamental human rights[3]. Instead of recommendations to modify these existing institutions, this paper focuses on improving efficiency by suggesting novel tax-neutral bargaining rules. While no bargaining efficient solution exists [10], there are numerous candidates for best alternative mechanism for contracting in this anticommon setting.

More specifically, this paper develops and compares several multiple agent bargaining rules. Of interest is how the predicted buyer and seller strategies change depending on the bargaining rule. From these predicted strategies, this paper generates predicted welfare results.

For the remainder of this paper, the term "contract rate" will refer to the probability that a transaction will occur conditional on the possibility of beneficial trade (i.e. there is some positive surplus available from trade) and "efficiency" will refer to percentage of captured surplus out of total possible surplus. The goal ultimately is to find the "best" rule that leads to the highest contract rate and efficiency[4].

2. Model

The simplest anticommon problem occurs when there is a single buyer who is interested in purchasing properties from two independent sellers. Assume that the buyer has no scrap value for obtaining only a single property. Thus the buyer has some non-negative package value,

[3] Universal Declaration of Human Rights – Article 17

$V_b \geq 0$. Each seller has only a single unit of a unique, indivisible property and some reservation cost for that property, respectively $V_1 \geq 0$ and $V_2 \geq 0$. If $V_b > V_1 + V_2$, then a positive surplus can be gained through exchange, and a mutually beneficial contract is possible. Let b (b_1 and b_2 for individual properties under the sequential non-contingent rule) be the buyer's "bid" for *both* properties. Let s_1 and s_2 be each seller's "ask" for his property. If $b > s_1 + s_2$ then a contract occurs and an exchange of properties for payment is made.

If a contract does occur, let P_b be the total amount that the buyer pays and P_1 and P_2 be the respective amounts that each seller receives. For a rule to be revenue neutral, $P_b = P_1 + P_2$. The exact amount paid and received will depend on the bargaining rule, b (or b_1 and b_2), s_1, and s_2. Additionally, let $\pi_b = V_b - P_b$ be the surplus that a buyer receives if he contracts. Let $\pi_1 = P_1 - V_1$ and $\pi_2 = P_2 - V_2$ be the surplus that each respective seller receives if he contracts. If no trades are made then $\pi_b = \pi_1 = \pi_2 = 0$.

2.1 Sequential Non-Contingent Ultimatum

To illustrate the efficiency loss caused by standard non-contingent bargaining and its corresponding potential for hold-up, consider the case of a "sequential ultimatum" rule. This sequential game consists of two separate ordered negotiations. The first bargaining stage always involves seller 1 and the buyer and the second bargaining stage (if it takes place) always involves seller 2 and the buyer. The second bargaining stage is conditional on contracting in the first stage, because the buyer only has value for the entire package of properties and consequently has no incentive to make a positive offer to the second seller after failing to obtain the first property. For each bargaining stage, a proposer places an ultimatum type offer to a responder. As in the standard ultimatum game[5], each responder may only accept or reject this offer. If a responder rejects an offer, then he may not propose nor hope for a counteroffer and no contract is reached.

If a responder accepts an offer, the two players trade at this proposed offer price. Let b_1, b_2, s_1, s_2 be proposal offers by the buyer in the first stage, the buyer in the second stage, the first seller, and the second seller respectively. So $P_1 = b_1$ or s_1 and $P_2 = b_2$ or s_2. Each player's respective payoffs are $\pi_b = V_b - (P_1 + P_2)$, $\pi_1 = P_1 - V_1$, and $\pi_2 = P_2 - V_2$ if both stages result in contracts. If the agents fail to contract in the first stage, $\pi_b = \pi_1 = \pi_2 = 0$. Finally, if only the first

[5] Güth et al. (1982)

stage results in a contract, $\pi_b = -P_1$, $\pi_1 = P_1 - V_1$, and $\pi_2 = 0$. With this last case, if the buyer purchases the first property but is unable to purchase the second property, then she receives negative profit.

2.2 Private Values

Part of the challenge in bargaining stems from each player's uncertainty regarding the value of the others.[6] Specifically, it is assumed that each player knows his or her own value, but only knows that the other players' values are independently drawn from distributions. Let f_1, f_2, and f_b be the respective density functions for V_1, V_2, and V_b. The respective distribution functions are F_1, F_2, and F_b with values $F_1(\underline{V}_1) = F_2(\underline{V}_2) = F_b(\underline{V}_b) = 0$ and $F_1(\overline{V}_1) = F_2(\overline{V}_2) = F_b(\overline{V}_b) = 1$. The expected profits of proposers (by round for the buyer) are:

$$\pi_1(s_1) = \int_{s_1}^{\overline{b}_1} (s_1 - V_1) g_{b_1}(b_1) db_1 \qquad (1a)$$

$$\pi_2(s_2) = \int_{s_2}^{\overline{V}_b} (s_2 - V_2) f_b(V_b) dV_b \qquad (1b)$$

$$\pi_b(b_1) = \int_{\underline{V}_2}^{b_2} \int_{\underline{V}_1}^{b_1} (V_b - b_1 - b_2) f_1(V_1) dV_1 f_2(V_2) dV_2 + \int_{b_2}^{\overline{V}_2} \int_{\underline{V}_1}^{b_1} (-b_1) f_1(V_1) dV_1 f_2(V_2) dV_2 \qquad (1c)$$

$$\pi_b(b_2 \mid b_1 \geq V_1) = -b_1 + \int_{\underline{V}_2}^{b_2} (V_b - b_2)) f_2(V_2)) dV_2 \qquad (1d)$$

Equations *(1a)* and *(1b)* state each respective seller's expected profits as functions of his ask and value and the buyer's density function, which is used to determine the expected probability of a contract. The g_{b_1} term in equation *(1a)* is not necessarily the same as f_b. For *(1a)* the first seller's expected probability of the responding buyer accepting his ask depends on the buyer's expectation of the second seller's ask. Let seller one's beliefs on the density of asks that the buyer will accept be $g_1(b_1)$. The buyer will accept any second stage ask that is below her value[7], but whether the buyer accepts a stage one ask is considerably more complicated.

Equations *(1c)* and *(1d)* respectively list the buyer proposing stage one and stage two expected payoffs. Each is a function of the buyer's value and her bid(s). Additionally, the first stage expected payoff also factors the probability of the second seller accepting a bid and the

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[6] The Myerson and Satterthwaite result is for bilateral bargaining with private values.
[7] By backward induction

second term factors in the possibility for the lost sunk cost. All four expected payoff equations assume that each agent only knows his or her own value.

As in the previous section, responders are assumed to accept offers that yield non-negative payoffs. If the buyer is the proposer in both periods, she will place offers to maximize her expected profits. To solve this sequential game, start with the last stage and by first using the fundamental theorem of calculus on *(1d)* to find the first order condition for b_2^*. The first order condition of *(1d)* with respect to b_2^* is $(V_b - b_2) f_2(b_2) - F_2(b_2) = 0$.

Even for this relatively simple case, the equilibrium strategies generated from the first order conditions may not be easily tractable or exist at all depending on the distributions of the values. Indeed, to obtain tractable results, results in the literature impose strong distributional assumptions in even the simpler bilateral bargaining case with incomplete information [4]. Not surprisingly, it will be necessary to impose similar distribution assumptions for this trilateral bargaining environment to generate tractable results for equilibrium strategies for this bargaining rule and the more complex ones discussed later in this paper.

More specifically, assume that V_1, V_2, and V_b are independently drawn from uniform distributions. Suppose $V_b \sim U[0,1]$, $V_1 \sim U[0,\frac{1}{2}]$, and $V_2 \sim U[0,\frac{1}{2}]$. That means that $f_1(x) = f_2(x) = 2$ for $x \in [0,\frac{1}{2}]$ and 0 otherwise and $f_b(x) = 1$ for $x \in [0,1]$ and 0 otherwise. The corresponding distribution functions are $F_1(x) = F_2(x) = 2x$ for $x \in [0,\frac{1}{2}]$, 0 for $x < 0$ and 1 for $x > \frac{1}{2}$ and $F_b(x) = x$ for $x \in [0,1]$, 0 for $x < 0$ and 1 for $x > 1$.

2.3. Non-contingent Buyer Proposes Ultimatum

Now with the distribution assumptions and expected profits equations from above, *(1c)* and *(1d),* finding a tractable solution to the non-contingent buyer propose ultimatum with incomplete information is possible.

THEOREM 1: *Given uniformly drawn agent values under the non-contingent "buyer proposes sequential ultimatum", the buyer will offer* $\frac{V_b^2}{4}$ *to the first seller and* $\frac{V_b}{2}$ *to the second seller, and the contract rate is only* $\frac{1}{6}$.

Using the distributional assumptions, the first order condition for *(1d)* is $V_b - 2b_2 = 0$. The second derivative for *(1d)* is less than 0 which confirms that $b_2^* = \frac{V_b}{2}$ is a maximum. To show that the buyer's optimal proposal to seller one is $b_1^* = \frac{V_b^2}{4}$, use backward induction to substitute $b_2^* = \frac{V_b}{2}$ into *(1c)* and then finding the first order condition with respect to b_1. More detailed explanations of these and other results found in this paper are in the appendix.

A rule is ex-post efficient if the probability of a mutually beneficial exchange occurring when possible is 1, and the probability of a transaction occurring that hurts all players is 0 [10]. Formally, ex-post efficiency (called contract rate here) for this particular "sequential ultimatum" case can be stated as this probability

$$\Pr((b_1^* \geq V_1) \cap (b_2^* \geq V_2) \mid V_b \geq V_1 + V_2). \qquad (2)$$

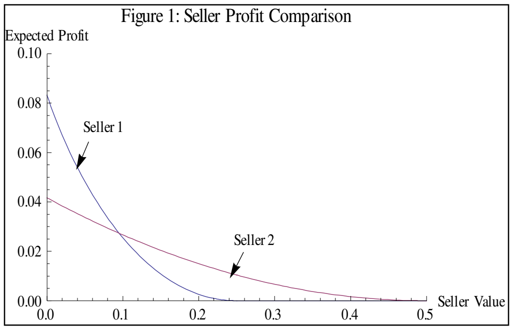
Based on the predicted buyer offers from the previous paragraph, both contracts are completed only $\frac{1}{6}^{th}$ of the time when there is available surplus. The corresponding buyer's expected profit is $\frac{V_b^4}{8} - \frac{V_b^5}{16}$. While it appears that the buyer still has all the bargaining power (that comes from being a first mover under an ultimatum rule), the incomplete information creates the possibility of a hold-up which severely decreases the total efficiency[8].

Seller one's corresponding expected profit is $\frac{1}{12} - V_1 + \frac{4V_1^{\frac{3}{2}}}{3}$ and seller two's corresponding expected profit is $\frac{V_2^2}{6} - \frac{V_2}{6} + \frac{1}{24}$. Because the buyer has some uncertainty regarding her value for the first property, the value is contingent on her ability to purchase the second property and the amount that she would have to pay for it. The second seller receives better bids from the buyer due to this uncertainty and the added bargaining power from the buyer's sunk purchase of seller one's property.

The second seller, however, may not receive more profit on average because the first seller must accept the first bid in order for the second seller to receive one. Figure 1 is provided to compare the expected profit difference between the sellers as functions of their respective values.

[8] If the buyer perfectly knew each seller's value, then the contract rate would be 1 (and thus the rule would be efficient) and the buyer would take the entire surplus.

Figure 2 illustrates the different bids each receives from the buyer as a function of the buyer's value.



The relatively low predicted contract rate is far from ideal. The buyer can be held-up because she is no longer certain of her ability to contract with the second seller. Due to the potential of being held-up, the buyer makes relatively low offers to the first seller which greatly decreases the probability of that first transaction occurring. This paper now turns to alternative contingent bargaining rules that prevent hold-ups from occurring and potentially improve welfare.

3. Simultaneous Offers

Ideally the buyer could form an agreement with the sellers to purchase a property only if she is able to purchase all the necessary complementary properties. If the buyer is able to negotiate a contingent contract with which both sellers agree, then this could drastically reduce the efficiency loss from potential hold-ups. In order for the sellers to accept these contingent contracts, the sellers would have to receive higher expected payoffs under contingent bargaining rules that under non-contingent ones (or have the buyer be willing to pay them in advance to negotiate under one).

First, consider a contingent simultaneous bargaining rule. Let the buyer make a single bid for the entire package of properties. This package bid prevents her from being held-up, and additionally allows her to clearly express her value for the entire package of properties. This is important because the buyer has difficulty evaluating her own value for just a single property when uncertainties exist regarding the price and probability of acquiring additional complementary ones. While the sellers do not appear to be directly bargaining with one another, in fact they are so because the buyer is only willing to pay a total fixed amount. Therefore, paying one additional dollar to one seller implies that the buyer has one less dollar to pay to the other seller.

Under this simultaneous sealed rule the buyer submits a single offer b and each seller submits a single ask, s_1 or s_2. If $b \geq s_1 + s_2$ then exchange occurs at some trade prices,

$P_b = h_b(b, s_1, s_2)$, $P_1 = h_1(b, s_1, s_2)$, and $P_2 = h_2(b, s_1, s_2)$. h is some pricing rule function that is used to determine the exact amount paid by the buyer and received by each seller. If no trade occurs then each player earns no profit. If trade occurs then $\pi_b = V_b - P_b$, $\pi_1 = P_1 - V_1$, and $\pi_2 = V_2 - P_2$. This h function greatly influences each agent's strategy.

3.1. Simultaneous Trilateral Bargaining Optimal Proportional Split

Expected gains from bilateral monopoly trade are maximized by Chatterjee and Samuelson's (1983) equal split bid mechanism [4][10]. The authors conclude that the optimal ex-post efficiency for two monopoly agents is .75. The impossibility result of an incentive compatible, individually rational, and ex-post efficient mechanism also applies to multilateral bargaining, but the optimal ex-post efficiency intuitively would be even lower due to the inherent bargaining difficulty of adding another self-interested party and the welfare maximizing surplus split rule may be correspondingly different.

Recall that the price paid by the buyer and received by each seller are respectively, $P_b = h_b(b, s_1, s_2)$, $P_1 = h_1(b, s_1, s_2)$, and $P_2 = h_2(b, s_1, s_2)$. To have a balanced budget price determination rule, $P_b = P_1 + P_2$. Ideally, $P_b = h_b(b, s_1, s_2) < b$, $P_1 = h_1(b, s_1, s_2) > s_1$, $P_2 = h_1(b, s_1, s_2) > s_2$ to ensure that each pays/receives no more than her bid/his ask. This ensures strong individual rationality for participation. Thus a bid would function as a maximum willing to pay and an ask would function as a minimum willing to receive. Additionally, each agent should get some proportion of the bargaining surplus. Define the this surplus as $S = b - s_1 - s_2$.

Let b be the maximum that the buyer states that she is willing to pay and the sum of the asks $(s_1 + s_2)$ is the lowest amount for which the buyer could have purchased the package. Thus, the buyer pays some amount, $P_b = kb + (1-k)(s_1 + s_2)$, where *k* is the proportion of the surplus that she does not receive. Given that the two sellers are identical in role, assume that they each receive the same proportion of surplus. Each seller *i* asks for s_i and could have asked for a maximum of $b - s_{-i}$ and still have contracted. Therefore, seller *i* should receive $P_i = (1-m)s_i + m(b - s_{-i})$, where *m* is the proportional of the surplus that the seller receives. If the exchange rule is revenue neutral, then $m = \frac{k}{2}$ must be true. The respective amounts paid and received are restated in terms of *k* are

$$P_b = kb + (1-k)(s_1 + s_2) \tag{4}$$

$$P_1 = \frac{k}{2}(b - s_2) + (1 - \frac{k}{2})(s_1)$$

$$P_2 = \frac{k}{2}(b - s_1) + (1 - \frac{k}{2})(s_2)$$

Given that all agents have incentive to bid and ask strategically, an agent's beliefs of opposing players' strategies must be considered. Let $s = s_1 + s_2$ be the total ask, $b_1 = b - s_2$ be the effective bid for seller one, and $b_2 = b - s_1$ be the effective bid for seller two. Furthermore let $g_b(s)$, $g_1(b_1)$, $g_2(b_2)$ respectively be the buyer's, seller one's and seller two's believed density of the total ask, effective bid one and effective bid two. In equilibrium, agents should optimally choose their strategy based on correct beliefs (i.e. the actual densities based on the other agents' strategies is the same as each agent's beliefs of them). Thus, $g_b(s) = f_s(s)$, $g_1(b_1) = f_{b_1}(b_1)$, and $g_2(b_2) = f_{b_2}(b_2)$. Each agent's expected profit is

$$\pi_b = \int_{s=\underline{s}}^{b} (V_b - kb - (1-k)s) g_b(s) ds \tag{5}$$

$$\pi_1 = \int_{b_1 = s_1}^{\overline{b_1}} (\frac{k}{2}(b_1) + (1 - \frac{k}{2})(s_1) - V_1) g_1(b_1) db_1$$

$$\pi_2 = \int_{b_2 = s_2}^{\overline{b_2}} (\frac{k}{2}(b_2) + (1 - \frac{k}{2})(s_2) - V_2) g_2(b_2) db_2$$

THEOREM 2: *Given random uniform agent values, under the family of offer strategies based on linear functions of value, the optimal split is when k is* $\frac{2}{3}$ *(equal surplus split). When all agents use the corresponding equilibrium strategies under this splitting rule, the contract rate is* $\frac{10}{27}$.

There are multiple Nash equilibria to this game. Any strategy profile that has all individual agents unable to improve their payoffs from single deviations is a Nash equilibrium. For example, with some available surplus, all strategy profiles that satisfy $b = s_1 + s_2$, $V_b \geq b$, $V_1 \leq s_1$, and $V_2 \leq s_2$ form a Nash equilibrium. Additionally there are trivial Nash equilibria such as the buyer bidding her minimum and each seller asking for the maximum, where no single agent can deviate to make a contract occur. This paper focuses on linear strategies of the form $b^*(V_b) = a_b V_b + k_b$ and $s_i^*(V_i) = a_s V_i + k_s$.

Assume that a buyer's optimal bid is some linear function of her value and each seller's optimal ask is some linear function of his value. The best response functions, which consider the combined densities of the other agents strategies ($g_b(s)$ - sum of two symmetric uniform distributions, $g_1(b_1)$ and $g_2(b_2)$ - differences of two asymmetric uniform distributions) will actually be linear in certain ranges. It is important that the best response functions are also linear in values, so that the beliefs match the actual strategies. By symmetry, it can be assumed that both sellers have the same best response functions.

Each agent has incentive to maximize his or her expected payoffs. Jointly solving (5a)-(5c) yields an equilibrium that has all agents mutually best responding. There are many combinations of mutual best responses, but there are the only a few linear "tractable" ones[9]. The corresponding equilibrium bid and asks strategies, listed as functions of value and *k* are listed in equation *(6).*

$$b^*(V_b) = \frac{2V_b}{2+k} + \frac{(2-k)k}{4(2+k)} \qquad (6)$$

$$s_i^*(V_i) = \frac{4V_i}{6-k} + \frac{(2-k)}{8}$$

Given that *k* determines the equilibrium strategies and these strategies determine the contract rate, this contract rate is some function of *k.* By simulation, a *k* value of approximately $\frac{2}{3}$ under case 1 leads to the highest contract rate of approximately .370. Substituting, $k=\frac{2}{3}$, equation *(6)* becomes

$$b^*(V_b) = \frac{3V_b}{4} + \frac{1}{12} \qquad (7)$$

$$s_i^*(V_i) = \frac{3V_i}{4} + \frac{1}{6}$$

Not surprisingly, $k=\frac{2}{3}$ is equivalent to an equal surplus split rule when there are 3 agents. Though this bargaining rule still results in efficiency loss, the approximate .370 contract rate is more than double the contract rate under the "buyer proposes sequential ultimatum" rule. Before

[9] See appendix 3.2 for derivations. Another linear case is found, but the "simpler" one of the two is shown and suffices for comparison between this bargaining rule and the others.

providing a detailed summary comparison of individual and group welfare under all discussed rules, one additional rule is considered.

3.2 Package Offer

Consider a bargaining rule where the buyer presents a "package ultimatum bid" to both sellers. In the first stage, let the buyer submit a package bid for both properties. Given that it is a contingent purchase rule, it precludes the possibility of a hold-up. In stage two, both sellers observe the buyer's package bid and submit asks. However, the buyer will now have an added incentive to understate her bid because this bid will be observed by the sellers.

This rule has nice properties for the buyer, specifically that it is a contingent rule. The buyer has some bargaining power in that she can credibly establish her position by making a bid and then let the seller's resolve the proceeding bargaining (i.e. the buyer has a first mover advantage). Additionally it may be more intuitive (easier for agents to understand) for practical applications than a single shot "simultaneous offer" rule since oftentimes it is the buyer that initiates action in these settings.

The second stage subgame between the sellers is equivalent to the classic Nash bargaining game. If the buyer bids at least as much as the sum of the sellers' asks, then any pair of asks (s_1, s_2) such that $b = s_1 + s_2$ is a Nash equilibrium of the subgame. As observed in the previous section, the surplus rule changes the equilibrium strategies and the corresponding predicted efficiency. One way of allocating this surplus is to find the maximum Nash product, which would allocate any surplus ($b-(s_1+s_2)$) evenly between the two sellers. This surplus rule is similar to the optimal split rule from section three and the optimal equal split bid rule from the bilateral monopoly literature [10]. Given this surplus split rule, each seller's second stage expected payoffs are

$$\pi_1 = \int_{s_1=\underline{s_1}}^{b-s_2} (s_1 + \frac{b-s_1-s_2}{2} - V_1) f_2(s_2) ds_2 \tag{9}$$

$$\pi_2 = \int_{s_2=\underline{s_2}}^{b-s_1} (s_2 + \frac{b-s_1-s_2}{2} - V_2) f_1(s_1) ds_1$$

By backward induction, in the first stage the buyer will offer a bid to maximize her expected payoffs:

$$\pi_b = \int_{s=\underline{s}}^{b} (V_b - b) f_s(s) ds, s = s_1^* + s_2^* \tag{10}$$

THEOREM 3: *Given random uniform agent values, the equilibrium strategies for a "package ultimatum offer" rule with an even second stage surplus split, lead to a contract rate of* $\frac{1}{3}$.

The subgame after the buyer makes her bid is equivalent to a standard Nash bargaining game with incomplete information. Each seller will ask for $s_i^* = \frac{b}{4} + \frac{2V_i}{3}$ in equilibrium and the buyer will correspondingly offer $b^* = \frac{2V_b}{3}$. Notice that the seller's equilibrium ask depends on his value and the buyer's bid. Ex-post efficiency or contract rate will be $\frac{1}{3}$ under these strategies. This is a higher contract rate than the "buyer proposes ultimatum" rule but still worse than the optimal sealed split rule.

This rule, however, may be more conducive for encouraging trade for certain value combinations and may have better expected payoffs for certain role and value combinations than the "optimal sealed split" rule. The following subsection analyzes the individual (by role) and group welfare for the "sequential ultimatum", "sealed optimal split", and "package offer" rules.

3.3 Individual and Group Welfare Summary:

The typical assumption is that a last seller would prefer a traditional sequential bargaining rule; the reason is that he could gain bargaining leverage by holding up the buyer. The "buyer proposes sequential ultimatum" section provides more evidence that the last seller earns more expected profit than the first seller. However, from section three there is evidence that the last seller would actually earn more expected profit under the simultaneous rule, even though it precludes a hold-up and thus removes his advantage of acting last.

Before entering bargaining, it may be that the agents can select their bargaining terms[10]. Agents with different values or costs and roles may prefer certain bargaining rules over others. For example, a buyer that earns higher expected profits under one rule compared to another may even be willing to pay up to the expected profit difference in order to bargain under the preferred rule.

This paper now proceeds to compare the three primary bargaining rules of interest in terms of individual welfare. The "buyer proposes sequential ultimatum" allows for a potential hold-up and serves as a baseline standard for welfare comparison. The "simultaneous optimal split" rule appears to benefit both buyers and sellers and is a candidate for the best multilateral

[10] E.g. the uniform commercial code allows for "exclusive negotiation rights" which is a specific case of a contractual agreement on bargaining terms but not an actual contract on terms of exchange.

bargaining rule in terms of contract rate. Finally, the “package ultimatum offer” appears to benefit the buyer as she can make a credible ultimatum bid and still not have to worry about a hold-up.

Ex-ante expected profits are listed for the buyer and seller (seller 2) below in Tables 1 and 2. These are the average expected profits if an agent knows his or her role and bargaining rule, but does not yet know his or her own value. This comparison is useful to determine the preferred rule for each role on average (regardless of value).[11]

[11] These figures are calculated based on simulations of the agents making “equilibrium” offers.

Bargaining Rule	***Ex-Ante Expected Profits***
Sequential Ultimatum	0.0250
Sealed Optimal Split	0.0320
Package Offer	0.0417

Table 1 - Buyer

Bargaining Rule	***Ex-Ante Expected Profits***
Sequential Ultimatum	0.0250 (0.0125)
Sealed Optimal Split	0.0320
Package Offer	0.2083

Table 2 – Seller 2(1)

Ex-ante profits are the best under the "package offer" for the buyer. The sellers' profits are best under the "simultaneous optimal split". The buyer places lower offers for the "package offer", because that offer is observed before the sellers make their asks. The buyer may not capture any of the surplus split because the "package ultimatum offer" is sequential, but she is able to take a first-mover advantage by establishing a credible position. Sellers should prefer the "sealed optimal split" over the "package ultimatum offer". The total ex-ante expected surplus is given in Table 3.

Bargaining Rule	***Total Ex-Ante Efficiency***	***Ex-Post Efficiency***
Sequential Ultimatum	0.0625	0.1667
Sealed Optimal Split	0.0960	0.3704
Package Offer	0.0833	0.3333

Table 3 – Combined (Seller 1 and 2 and Buyer)

The sealed bargaining rule leads agents to contract the most frequently when beneficial to do so and leads to the highest ex-ante efficiency. However, if a rule has the highest ex-ante efficiency, this does not mean necessarily mean that it must have the highest contract rate. This discrepancy is possible if a rule leads to failure in contracting when larger surpluses are available.

The sequential nature of the "package offer" means that sellers receive some buyer's package value information before they make their asks. The "sealed optimal split" rule leads to relatively equitable profits though between the buyer and the two sellers, while the buyer receives a larger share of the surplus in the "package offer". Figures 3 and 4 respectively compare the expected profits for the buyer and the seller (seller 2 for sequential) under the buyer proposes ultimatum, the simultaneous sealed split bid, and the "package offer".

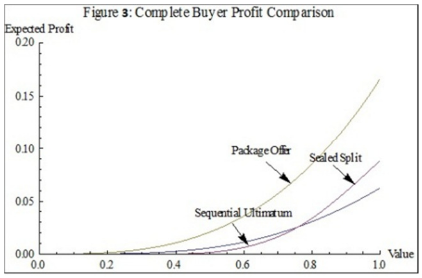


Figure 3: Complete Buyer Profit Comparison

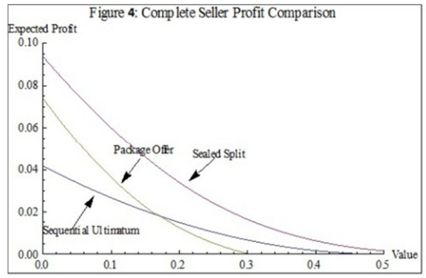


Figure 4: Complete Seller Profit Comparison

The buyer prefers the "package offer" for all values as compared to the "sealed optimal split" or the "sequential ultimatum". Under this rule, the buyer is not held-up and can make an ultimatum type offer. Even though the buyer has some bargaining power in the non-contingent sequential case that comes from her ability to make the ultimatum bids, the buyer prefers the sealed split rule for higher values. The sellers prefer the "sealed split" for all values. For low seller values and high buyer values, all agents prefer either of the contingent contract rules to the sequential ultimatums, so the agents would agree to bargain under either the "simultaneous sealed split" or "packaged ultimatum offer". The seller has clear incentive to choose the "sealed split" rule over the "sequential ultimatum" despite the perceived bargaining advantage of being the last seller.

Even if some agents actually decrease their expected payoffs from switching from the "sequential ultimatum" to either contingent rule, the overall welfare increase allows the welfare gaining agent to be able to compensate the other agents before the bargaining regime is decided and still manage to increase his or her expected profits.

4. Experimental Design and Predictions

The purpose of this section is to determine whether the theoretical findings from adequately reflect actual behavior. To properly address this question, these theoretical results were tested in a laboratory experiment. The strategy and welfare predictions for all the aforementioned bargaining rules will be compared to the actions and welfare from groups of human subjects who bargain over fictitious properties with predetermined assigned values under these rules.

The experiments consisted of multiple sessions of four different bargaining rule treatments. Three of treatments are the rules that are described in the previous sections, "sequential ultimatum", the "sealed optimal split", and "package offer". The final rule is a continuous unrestricted three way auction, "*triple auction*". This rule can be modeled as a finitely repeated game, where in the stage game each player can submit an offer or alternatively choose to pass. If by the deadline, the standing bid does not exceed the sum of the asks then no transaction occurs. If at anytime, the standing bid exceeds the standing sum of asks then a transaction immediately occurs and each player will get her bid or his ask.

Each session consists of forty rounds of a single treatment and had groups ranging in size from 12 to 24 subjects (always a multiple of 3 for grouping purposes to match the single buyer and two sellers). Forty rounds was chosen as compromise to allow for a larger sample size to provide adequate opportunities for contracting, but to still limit subject fatigue as roughly a minute was allotted for each round (approximately an hour total per session including practice and instructions). Table 1 provides the total number of sessions and groups (of 3) for all sessions combined for each treatment.

Treatment	**Sessions**	**Groups**
Sequential Ultimatum	7	36
Sealed Optimal Split	7	40
Package Offer	5	30
Triple Auction	6	27

Table 1: Treatments Summary

At the start of each session, subjects read computerized instructions (see Appendix B), took a brief quiz regarding the particular bargaining rule of their session to verify that subjects could calculate profits given a set of values, bids, and asks, and had several practice rounds of the treatment's bargaining rule. At the beginning of each round, subjects were assigned to a group and a role in that group with a corresponding random uniform pre-drawn value ranging from 0 to 100 for the buyer and 0 to 50 for a seller. The whole numbers could have potentially created a minor discreteness issue, but their usage facilitated subject navigation of the environment.

Besides timing restrictions to expedite rounds, the bargaining rules under each treatment were identical to their corresponding rules described above in section 2. Subjects had 45 seconds total for the "simultaneous offer", 15 seconds for each offer for the sequential ultimatum, 30 seconds for each offer for the "buyer package offer", and 1 minute total for the "triple auction". The design was to allow time for subjects to strategize and submit their offers but to limit it, so that subjects do not become disengaged.

Also, the triple auction, due to the repeated offers, required additional information to each player. Because theoretically a solely self-regarding rational agent should not care about an individual subject's value (it is always the combination of the other two values which determines the available surplus), subjects could not see specific offers made by others. Subjects, however, did observe the "effective bid" or "total ask". Essentially, each player observed the combination of the offers made by the other players but neither individual player's offer. The buyer observes information regarding the total ask but not either individual ask. Each seller observes information regarding the effective bid (the buyer's bid minus the other seller's ask), but not the actual bid or other ask. This information set was also chosen in part to streamline the viewing area for subjects.

While showing this information instead of all the individual bid and asks does not make a difference according to the standard rational agent economic model, behavioral elements such as spite could play a role in decision making if agents felt another identifiable individual agent was being non-cooperative. This is problem is mitigated in part by random and anonymous groupings.

The experiments were conducted at computer terminals at a local university. In all sessions, after each round subjects receive information regarding the previous bid, asks, contract completion and their individual profits. For smaller sessions there is some concern that subjects will adjust strategies based on these repeated interactions. Over the course of 40 rounds, especially for these smaller sessions, it becomes increasingly likely that there could be some autocorrelation due to repeated interactions even though the roles, values, and groups are changed (and randomly formed) each round. Unfortunately, the experiment lacks the leverage to distinguish this effect from exhibited learning.

The values for all sessions are the same, but the bargaining rule and corresponding order of actions varies. The values must be the same across treatments to observe the treatment effect. From these values, there are potential gains from trade for twenty two out of forty sessions. There was some concern that subjects would be discontent dealing with such high potential failure (in terms of not reaching a contract) rates. However, because the theoretical results from this paper are based on values that are random uniform drawn, there will be substantial contract failure even under full efficiency. Yet it is necessary to have comparable experimental values to preserve the integrity of the comparison between the experimental results and theoretical predictions.

Table 2 lists the predicted percentage of total contracts, total beneficial contracts, and surplus out of all possible surplus (% efficiency) based on the drawn values for the 40 rounds if all agents behaved according to their equilibrium strategies.

Institution	**Predicted % Contracts**	**Predicted % Contracts Rate**	**Predicted % Efficiency**
Sequential Ultimatum	0.15	0.273	0.46
Sealed Optimal Split	0.25	0.455	0.732
Package Offer	0.3	0.545	0.813

Triple Auction	*0.25*	*0.455*	*0.732*

Table 2: Predicted Group Welfare

These predictions are based on all subjects using the predicted theoretical equilibrium strategies. The "triple auction" predictions are identical to the predictions for the "sealed optimal split"[12]. For these 40 particular values, the "buyer package offer" is actually predicted to lead to a slightly higher contract rate and efficiency than the "sealed optimal split". Next, Table 3 shows the individual welfare predictions under these values for each treatment.

Institution	**Predicted Buyer Profit**	**Predicted Seller 1 Profit**	**Predicted Seller 2 Profit**
Sequential Ultimatum	*138*	*66*	*150*
Sealed Optimal Split	177	193	193
Package Offer	302	161.5	161.5
Triple Auction	*177*	*193*	*193*

Table 3: Predicted Individual Welfare

Sequential ultimatum is predicted to do the worse for all three roles. In particular, the first seller role is predicted to do very poorly, because the buyer is predicted to make low first bids. Subjects in all roles are predicted to do better under any contingent trade rule than under the separate rule.

5. Empirical Results

At the group level, the theoretical welfare predictions turned out to be fairly accurate. The order of predicted rule performance is mostly upheld. Subjects performed better than predicted under some rules and worse than predicted in others. More specifically, subjects fared the best under the "sealed optimal split" and the "triple auction", exceeding predictions of both contract rate and efficiency. On the other hand, subjects performed worse than predicted under "sequential ultimatum" and "package offer". In particular, subjects performed well below prediction for the "package offer" rule. Table 4 summarizes the group welfare results.

Institution	**Sequential Ultimatum**	**Sealed Optimal**	**Package Offer**	**Triple Auction**

[12] If holdout incentives are considered then subjects will delay offers until the deadline (because any non-deadline offer weakens bargaining power by giving others value information and no player has any incentive to accept a non-deadline offer because it can always be improved) so that this repeated sequential game will effectively degenerate into a simultaneous offer gam[7][11][12].

		Split		
Total Sample Size	1440	1600	1200	1080
Contracts	335	545	356	493
Beneficial Contracts	323	533	337	429
%Contracts	0.233	0.341	0.297	0.456
Contract Rate	0.408	0.606	0.511	0.722
Possible Surplus	27684	30760	23070	20763
Actual Profit	10301	23412	14720	16318
%Efficiency	0.372	0.761	0.638	0.786

Table 4: Group Summary Data[13]

For three of the four treatments, the expected contract percentage and the expected contract when beneficial are significantly higher than expected. (Note that for the "sequential ultimatum", contracts are only counted when the buyer is able to purchase both properties.) One possible explanation is that subjects may feel that they are supposed to contract and may actually derive some inherent utility from contracting in itself. More surprisingly, the "package offer", which is predicted to have the highest completed contract rate, fared significantly worse than both the "sealed optimal split" and the "triple auction".

RESULT 1: *Overall, subjects had the most welfare under the "sealed optimal split" and the "triple auction".*

The theoretical predictions for the "sealed optimal split" is that the rule leads to a higher contract rate and ex-ante efficiency than the "package offer" when including all possible value draws from the stated distributions. However, the predictions based on the specific experimental value set suggest that the "package offer" will lead to a higher contract rate and efficiency. Yet, the "sealed optimal split" did substantially better both in terms of contract rate and efficiency. "Package offer" results will later be discussed in greater depth in the individual welfare section.

RESULT 2: *The "triple auction" had different welfare results from the "sealed simultaneous optimal split".*

It is expected that the "triple auction" equilibrium strategies and results will degenerate to those of the sealed optimal split. Subjects are predicted not to make bids or asks until the last possible time increment. (First offers were actually made in an average 8 seconds out of a possible 60 seconds.) The "triple auction" resulted in only a slightly higher percentage of efficiency despite a much higher percentage of beneficial contracts. This suggests that much of

[13] The large sample sizes suggest that the differences are statistically significant.

the same achievable efficiency was already gained in bargaining under the “sealed optimal split”. However, the contract rate difference suggests that the two rules are not identical. If holdout incentives are weak, then subjects may capture additional surplus from extra information learned through sequential bargaining.

RESULT 3: *Subjects had significantly higher efficiency under any of the contingent contract rules than the under sequential rule.*

The “sequential ultimatum” treatment had the lowest beneficial contract completion percentage by more than 10 raw percentage points. This rule did even worse in terms of the percentage of captured total surplus. It lead to 26% less captured surplus than any of the contingent contract rules. Implying that even switching to the worst contingent alternative led to more than a 70% increase in efficiency! This low efficiency result provides very strong empirical evidence that given a choice of bargaining rule that subjects should prefer a contingent rule to a standard sequential acquisition one.

4.1 Individual Welfare Results

The individual welfare results, unlike the group ones, differ substantially from the theoretical predictions. Although as groups, subjects contracted when beneficial more than predicted, overall subjects had lower profits than predicted, which is likely due in large part to the irrational trades. The differences between the predictions for each role under each rule vary even more drastically. Table 5 lists the profits for each role over all 40 rounds and the difference between these profits and the predicted profits from table 3.

Institution	**Sequential Ultimatum**	**Sealed Optimal Split**	**Package Offer**	**Triple Auction**
Buyer Profit	-165.556	264.763	118.3	197.296
Seller 1 Profit	307.111	156.694	177.75	201.111
Seller 2 Profit	144.583	163.844	194.617	205.963
Buyer difference	-303.556	87.762	-183.7	20.296
Seller 1 difference	241.111	-36.306	16.25	8.111
Seller 2 difference	-5.417	-29.156	33.117	12.963

Table 5: Individual Profits (per 40 rounds)

The first thing to notice is that the baseline treatment of “sequential ultimatum” results in *losses* to buyers. From the data, it is clear that buyers are bidding far too much (as compared to predicted optimal behavior) for the first property. This results in large losses due to incomplete projects. The second sellers, who are expected to receive better offers and earn more due to the hold up, actually do not perform nearly as well because they do not receive nearly as many offers.

So, rather than initially under-investing as predicted theoretically, buyers suffer losses and first sellers reap the benefit. Note, this finding strongly contradicts the standard assumption of late bargaining advantage and initial underinvestment in hold-up settings.

The level of sophistication required of buyers in this setting is non-trivial because it can be challenging to calculate the proper offer for a property that has no value by itself but has some package value if acquired with additional complementary properties.

The buyers received much more profit than predicted for the "simultaneous offer" but much less than predicted for the "package offer". The sellers, rather than the buyer, actually earned more on average under the "package offer" than the "simultaneous offer". At the individual welfare level, the "triple auction" rule is the only rule that yielded results that are similar to the predictions.

5. Conclusions:

Overall, there is compelling theoretic and empirical evidence that contingent bargaining rules lead to more beneficial contracts and higher total welfare. There are numerous contract and surplus rules including the ones discussed in this paper that could potentially facilitate agent coordination in these challenging anticommon environments.

While some of the empirical findings contradicted the theoretical predictions at the individual welfare level, there is still significant empirical evidence that contingent bargaining rules lead to more beneficial contracts and higher total welfare. Specifically, there is empirical support for the triple auction and sealed optimal split rules as being strong alternatives to sequential bargaining.

Appendix A: Proofs
Section 2.3
In theorem 1, the buyer's expected profit must be maximized at each stage.
Substitute the buyer's second stage offer into equation *(1c)* to find the first stage profit,

$$\pi_b(b_1) = \int_{V_2=0}^{\frac{V_b}{2}} \int_{V_1=0}^{b_1} ((V_b - b_1 - \frac{V_b}{2}))4dV_1dV_2 + \int_{V_2=\frac{V_b}{2}}^{\frac{1}{2}} \int_{V_1=0}^{b_1} (-b_1)4dV_1dV_2$$

Integrate with respect to V_2,

$$\pi_b(b_1) = 2\int_0^{b_1} V_b(\frac{V_b}{2} - b_1)dV_1 - 2\int_0^{b_1} (1-V_b)b_1dV_1$$

and V_1,

$$\pi_b(b_1) = V_b^{\ 2}b_1 - 2b_1^{\ 2}$$

yields an expected profit function as a function of constants and the first stage bid.
Find the first order condition to maximize profit.
The derivative with respect to b_1 is

$$\frac{\partial \pi_b}{\partial b_1} = V_b^{\ 2} - 4b_1.$$

Setting this equal to 0 gives

$$b_1^{\ *} = \frac{V_b^{\ 2}}{4}.$$

The second order condition,

$$\frac{\partial^2 \pi_b}{\partial b_1^{\ 2}} : -4 < 0 \quad \forall V_b \in [0,1]$$

confirms that $b_1^{\ *} = \frac{V_b^{\ 2}}{4}$ is a local max.
In order to find the beneficial contract rate, must find the conditional probability.
If $b_1 \geq V_1 \cap b_2 \geq V_2$ and $V_b < V_1 + V_2$ then $\frac{V_b^{\ 2}}{4} + \frac{V_b}{2} \geq V_1 + V_2 > V_b$, so $\frac{V_b^{\ 2}}{4} - \frac{V_b}{2} > V_b$.
However this statement can be only true if $V_b = 0$ or if $V_b > 2$, (which never leads to a transaction), which means that $\Pr(b_1 \geq V_1 \cap b_2 \geq V_2 \mid V_b < V_1 + V_2) = 0$
This means that the event of contracting is a subset of the event when surplus is available.
So the conditional probability must equal $\frac{\Pr(b_1 \geq V_1 \cap b_2 \geq V_2)}{\Pr(V_b \geq V_1 + V)}$.
The numerator for this probability can be rewritten as
$\Pr(b_1 \geq V_1 \cap b_2 \geq V_2 \cap V_b \geq V_1 + V_2) = \Pr((b_1 \geq V_1 \cap b_2 \geq V_2) = \Pr(b_1 \geq V_1)\Pr(b_2 \geq V_2 \mid b_1 \geq V_2)$.
The probability of the buyer winning the first property is

$$\Pr(b_1 \geq V_1) = \int_{V_b=0}^{1} \int_{V_1=0}^{\frac{V_b^{\ 2}}{4}} f_1(V_1)f_b(V_b)dV_1dV_b = \frac{1}{6}.$$

Given that the buyer won the first property, the probability of her winning the second is

$$\Pr(b_2 \ge V_2 \mid b_1 \ge V_1) = \int_{V_b=0}^{1} \int_{V_2=0}^{\frac{V_b}{2}} f_2(V_2) f_b(V_b) dV_2 dV_b = \frac{1}{2}.$$

The probability of any surplus is

$$\Pr(V_b \ge V_1 + V_2) = \int_{V_b=0}^{1} \int_{V_1+V_2=0}^{V_b} f_{1+2}(V_1+V_2) f_b(V_b) d(V_1+V_2) dV_b.$$

$V_1 + V_2$ is the sum of two variables distributed $U[0,\frac{1}{2}]$ so the pdf for the sum (using the convolution theorem for sums) is

$$f_{1+2}(x) = \begin{array}{c} (4x), x \in [0,\frac{1}{2}] \\ (4-4x), x \in (\frac{1}{2},1] \\ 0, otherwise \end{array}.$$

Evaluating the probability of surplus,

$$\int_{V_b=0}^{\frac{1}{2}} \int_{V_1+V_2=0}^{V_b} 4(V_1+V_2)d(V_1+V_2)dV_b + \int_{V_b=\frac{1}{2}}^{1} [\int_{V_1+V_2=0}^{\frac{1}{2}} 4(V_1+V_2)d(V_1+V_2) + \int_{V_1+V_2=\frac{1}{2}}^{V_b} 4-4(V_1+V_2)d(V_1+V_2)]dV_b = \frac{1}{2}$$

So the contract rate then is

$$\Pr(b_1 \ge V_1 \cap b_2 \ge V_2 \mid V_b \ge V_1 + V_2) = \frac{\Pr(b_1 \ge V_1)\Pr(b_2 \ge V_2 \mid b_1 \ge V_1)}{\Pr(V_b \ge V_1 + V_2)} = \frac{1}{6}.$$

The expected profit for the buyer given his equilibrium strategies is

$$\pi_b(b_1^*, b_2^*) = 4\int_{v_2=0}^{\frac{V_b}{2}} \int_{v_1=0}^{\frac{V_b^2}{4}} (\frac{V_b}{2} - \frac{V_b^2}{4}) dV_1 dV_2 - 4\int_{v_2=\frac{V_b}{2}}^{\frac{1}{2}} \int_{V_1=0}^{\frac{V_b^2}{4}} (\frac{V_b^2}{4}) dV_1 dV_2 = \frac{V_b^4}{8} - \frac{V_b^5}{16}.$$

This satisfies the participation constraint because $\frac{V_b^4}{8} - \frac{V_b^5}{16} > 0$ for all $V_b \in [0,1]$.

The second seller's expected profit is

$$\pi_2 = \Pr(b_1^* \ge V_1) \int_{b_2=V_2}^{\frac{1}{2}} (b_2 - V_2) f_{b_2}(b_2) db_2 = \frac{V_2^2}{6} - \frac{V_2}{6} + \frac{1}{24}.$$

The corresponding pdf of the buyer's second bid is $f_{b_2}(b_2) = 2$ for $b_2 \in [0,\frac{1}{2}]$, 0 otherwise.

The first seller's expected profit is

$$\pi_1 = \int_{b_1=V_1}^{\frac{1}{4}} (b_1 - V_1) f_{b_1}(b_1) db_1.$$

Given that $b_1^* = \left(\frac{V_b}{2}\right)^2$ and $f_{b_1}(b_1) = \frac{1}{\sqrt{b_1}}$ for $0 < b_1 \le \frac{1}{4}$, 0 otherwise,

$$\pi_1 = \int_{b_1=V_1}^{\frac{1}{4}} (b_1 - V_1)\frac{1}{\sqrt{b_1}} db_1 = \frac{1}{12} - V_1 + \frac{4V_1^{\frac{3}{2}}}{3}.$$

Section 3.1

In theorem 2, the buyer and sellers must each choose her or his profit maximizing offer. Given that an agent's offer is a linear functions of her or his value, $b^*(V_b) = a_b V_b + k_b$ and $s_i^*(V_i) = a_s V_i + k_s$.

Because there are two identical sellers, consider only symmetric solutions.

The buyer's correctly believes that the pdf of the total ask is

$$g_b(s) = \begin{cases} \frac{4s - 8k_s}{a_s^2}, s \in [2k_s, 2k_s + \frac{a_s}{2}] \\ \frac{-4s + 8k_s + 4a_s}{a_s^2}, s \in (2k_s + \frac{a_s}{2}, 2k_s + a_s]. \\ 0, otherwise \end{cases}$$

And each seller correctly believes that the pdf of the effective bid that he will receive is

$$g_i(b_i) = \begin{cases} \frac{2(b_i + k_s + \frac{a_s}{2} - k_b)}{a_b a_s}, b_i \in [k_b - k_s - \frac{a_s}{2}, k_b - k_s) \\ \frac{1}{a_b}, b_i \in [k_b - k_s, k_b - k_s + a_b - \frac{a_s}{2}) \\ \frac{2(-b_i - k_s + k_b + a_b)}{a_b a_s}, b_i \in [k_b - k_s + a_b - \frac{a_s}{2}, k_b - k_s + a_b] \\ 0, otherwise \end{cases}.$$

If the buyer's optimal bid is $b \le 2k_s + \frac{a_s}{2}$ then the buyer's expected profit is

$$\pi_b = \int_{s=2k_s}^{b} (V_b - kb - (1-k)s)(\frac{4s - 8k_s}{a_s^2}) ds. \qquad \textit{(A1)}$$

If instead the buyer's optimal bid is $b > 2k_s + \frac{a_s}{2}$ then the buyer's expected profit is

$$\pi_b = \int_{s=2k_s}^{2k_s + \frac{a_s}{2}} (V_b - kb - (1-k)s)(\frac{4s - 8k_s}{a_s^2}) ds + \int_{s=2k_s + \frac{a_s}{2}}^{b} (V_b - kb - (1-k)s)(\frac{-4s + 8k_s + 4a_s}{a_s^2}) ds.$$

Label the above equation *(A2)* to represent the "higher" optimal bid case.

For each seller, if $k_b - k_s + a_b - a_s < s_i$ then the expected seller profit is

$$\pi_i = \int_{b_i = s_i}^{k_b - k_s + a_b} (\frac{k}{2}(b_i) + (1 - \frac{k}{2})(s_i) - V_i) \frac{2(-b_i - k_s + k_b + a_b)}{a_b a_s} db_i. \qquad \textit{(A3)}$$

If instead $k_b - k_s < s_i \le k_b - k_s + a_b - a_s$, then the expected seller profit is

$$\pi_i = \int_{b_i = s_i}^{k_b - k_s + a_b - \frac{a_s}{2}} (\frac{k}{2}(b_i) + (1 - \frac{k}{2})(s_i) - V_i)\frac{1}{a_b} db_i +$$

$$\int_{b_i = k_b - k_s + a_b - \frac{a_s}{2}}^{k_b - k_s + a_b} (\frac{k}{2}(b_i) + (1 - \frac{k}{2})(s_i) - V_i)\frac{2(-b_i - k_s + k_b + a_b)}{a_b a_s} db_i \,. \qquad (A4)$$

Finally if $s_i \leq k_b - k_s$, then the expected seller's profit is

$$\pi_i = \int_{b_i = s_i}^{k_b - k_s} (\frac{k}{2}(b_i) + (1 - \frac{k}{2})(s_i) - V_i)\frac{2(b_i + k_s + \frac{a_s}{2} - k_b)}{a_b a_s} db_i +$$

$$\int_{b_i = k_b - k_s}^{k_b - k_s + a_b - \frac{a_s}{2}} (\frac{k}{2}(b_i) + (1 - \frac{k}{2})(s_i) - V_i)\frac{1}{a_b} db_i +$$

$$\int_{b_i = k_b - k_s + a_b - a_s}^{k_b - k_s + a_b} (\frac{k}{2}(b_i) + (1 - \frac{k}{2})(s_i) - V_i)\frac{2(-b_i - k_s + k_b + a_b)}{a_b a_s} db_i \,. \qquad (A5)$$

There are best responses for both buyer's cases and all three seller cases.
However, if offers are linear functions of values, then the best response offers must also be linear strategies in order for the beliefs to be correct and for the equilibrium best response strategies to be linear.
Next, take find the optimal offers by maximizing profits with respect to the offer.
The optimal offer from the first order condition for *(A1) is*

$$b^* = \frac{2kk_s}{2+k} + \frac{2V_b}{2+k}.$$

The optimal offer from the first order condition for *(A2)* is

$$b^* = \frac{2a_s + 2a_s k + 4k_s + 4kk_s + 2V_b \pm (\sqrt{2})\sqrt{2a_s^2 + 2a_s^2 k + 2a_s^2 k^2 + 8a_s k_s + 8k_s^2 - 4a_s V_b - 8k_s V_b + 2V_b^2}}{2(2+k)}.$$

Only the first buyer's case is a linear function of value.
The seller's optimal offer from the first order condition for *(A3)* is

$$s_i^* = \frac{-2a_b + a_b k - 2k_b + kk_b + 2k_s - kk_s}{-6+k} + \frac{-4V_i}{-6+k}.$$

The seller's optimal offer from the first order condition for *(A4)* is

$$s_i^* = \frac{-8a_b + 2a_s + 4a_b k - a_s k - 8k_b + 4kk_b + 8k_s - 4kk_s}{4(-4+k)} + \frac{-8V_i}{4(-4+k)}.$$

The seller's optimal offer from the first order condition for *(A5)* is

$$s_i^* = \frac{1}{2(-6+k)}[(8a_s - 2a_s k - 8k_b + 2kk_b + 8k_s - 2kk_s - 4V_i) \pm$$

$$\sqrt{48a_b a_s + 52a_s^2 - 32a_b a_s k - 24a_s^2 k + 4a_b a_s k^2 + 3a_s^2 k^2 - 32a_s k_b + 16k_b^2 + 32a_s k_s - 32k_b k_s + 16k_s^2 + 32a_s V_i - 32k_b V_i + 32k_s V_i + 16V_i^2}\,].$$

Only the first two seller's cases are linear functions of value.
In order to find the equilibrium strategies, jointly solving (A1) and (A3) simultaneously. Also use the linear functional assumptions, $s_i^* = a_s V_i + k_s$ and $b^* = a_b V_b + k_b$ with the equilibrium strategies to find the parameter values.

For the seller, the parameter values are

$$a_s = \frac{-4}{-6+k},\ k_s = \frac{2-k}{8}.$$

And for the buyer, the parameter values are

$$a_b = \frac{2}{2+k},\ k_b = \frac{-k(-2+k)}{4(2+k)}.$$

The corresponding equilibrium linear strategies are

$$s_i^*(V_i) = \frac{-4V_i}{-6+k} + \frac{2-k}{8},\ b^*(V_b) = \frac{2V_b}{2+k} + \frac{-k(-2+k)}{4(2+k)}.$$

By applying the same technique to the combination of *(A1)* and *(A4),* the seller parameter values are

$$a_s = \frac{-2}{-4+k},\ k_s = \frac{-(-28+24k-5k^2)}{4(-6+k)(-4+k)}.$$

And the buyer parameter values are

$$a_b = \frac{2}{2+k},\ k_b = \frac{k(28-24k+5k^2)}{2(48+4k-8k^2+k^3)}.$$

The corresponding equilibrium strategies are

$$s_i^*(V_i) = \frac{-2V_i}{-4+k} + \frac{-(-28+24k-5k^2)}{4(-6+k)(-4+k)}$$

$$b^*(V_b) = \frac{2V_b}{2+k} + \frac{k(28-24k+5k^2)}{2(48+4k-8k^2+k^3)}.$$

Contract rate is some function of *k,*

$$\Pr\left(\left(\frac{2V_b}{2+k} + \frac{-k(-2+k)}{4(2+k)} \geq \frac{-4(V_1+V_2)}{-6+k} + \frac{2-k}{4}\right) \mid V_b \geq V_1+V_2\right).$$

This probability is maximized when $k = \frac{2}{3}$.[14]

The $k = \frac{2}{3}$ is analogous to the "split bid" rule in bilateral negotiations.

Note that if $k = \frac{2}{3}$ then the equilibrium offers are $b^* = \frac{3V_b}{4} + \frac{1}{12}$ and $s_i^* = \frac{3V_i}{4} + \frac{1}{6}$.

The corresponding contract rate is $\Pr((V_b - \frac{1}{3} \geq V_1+V_2) \mid V_b \geq V_1+V_2)$.

$\Pr((V_b - \frac{1}{3} \geq V_1+V_2)$ is equal to

$$\int_{\frac{1}{3}}^{\frac{2}{3}} \int_0^{V_b-\frac{1}{3}} 4V_s dV_s dV_b + \int_{\frac{2}{3}}^{1} [\int_0^{\frac{1}{2}} 4V_s dV_s + \int_{\frac{1}{2}}^{V_b-\frac{1}{3}} (4-4V_s) dV_s] dV_b = \frac{5}{27}.$$

[14] To analytically show this result, first simplify and then take the derivative of the corresponding probability value with respect to *k* to solve for *k.* This result has been verified through simulation.

Because $\Pr(V_b \geq V_1 + V_2) = \frac{1}{2}$, the corresponding contract rate then is $\frac{10}{27}$.

Section 3.2

In theorem 3, each seller's best response depends on his value and the observed offer that is made by the buyer, so let $s_i^* = a_b b + a_s V_i + k_s$.

Take the derivative of equation *(8)* with respect to the seller offer to find that the seller's optimal ask is

$$s_i^* = \frac{b - a_b b - k_s + 2V_1}{3}.$$

This implies that the parameters are $a_b = \frac{1}{4}, a_s = \frac{1}{3}, k_s = \frac{-1}{3}$. The other seller's best response to this is

$$s_j^* = \frac{1}{4}b + \frac{2}{3}V_i + \frac{1}{9}k_s.$$

And the best response to this is

$$s_j^* = \frac{1}{4}b + \frac{2}{3}V_i - \frac{1}{27}k_s.$$

In order for a strategy profile to be a Nash equilibrium, each seller's strategy must be a best response to the other seller's. Notice that by iteratively finding the best response the coefficient on this k_s term approaches 0.

If this coefficient is 0, the best response is

$$s_j^* = \frac{1}{4}b + \frac{2}{3}V_j.$$

Player i's best response to this is

$$s_i^* = \frac{1}{4}b + \frac{2}{3}V_i.$$

Finally, there is a mutual best response correspondence and a Nash equilibrium is found.

In order for the buyer to determine her optimal bid she must use backward induction, because her bid affects the sellers' asks. The pdf of the sum of the sellers' asks is

$$f_s(s) = \begin{matrix} 9s - \frac{9b}{2}, s \in [\frac{b}{2}, \frac{b}{2} + \frac{1}{3}) \\ -9s + \frac{9b}{2} + 6, s \in [\frac{b}{2} + \frac{1}{3}, \frac{b}{2} + \frac{2}{3}] \end{matrix}.$$

The buyer's expected profit (from her offer in the first stage is)

$$\pi_b \int_{s=\frac{b}{2}}^{b} (V_b - b) f_s(s) ds.$$

Solving for the first order condition,

$$b^* = \frac{2V_b}{3}.$$

The beneficial contract rate is the same conditional probability with these new equilibrium offers. The $\Pr(b^* \geq s_1^* + s_2^*)$ is

$$\Pr(\frac{2}{3}V_b \geq \frac{1}{2}b + \frac{2}{3}(V_1 + V_2)) .$$

Simplifying this and evaluating,

$$\Pr(\frac{V_b}{2} \geq V_1 + V_2) = \frac{1}{6} .$$

So the beneficial contract rate, $\Pr(b^* \geq s_1{}^* + s_2{}^* \mid V_b \geq V_1 + V_2) = \frac{\Pr(b^* \geq s_1{}^* + s_2{}^*)}{\Pr(V_b \geq V_1 + V_2)} = \frac{1}{3}$.

Appendix B: Instructions

General Experiment Instructions

In this experiment you will participate as either a buyer or seller. Each group will consist of 1 buyer and 2 sellers. Your role will alternate randomly throughout the experiment. There will be 40 rounds in this experiment. In each round, you will be randomly grouped and assigned the role of a buyer or seller.

All earnings will be in experimental dollars which will be converted to U.S. Dollars at the end of the experiment.

Each round, you will be assigned a new role and corresponding value. Buyers have package values that are randomly drawn between 0 and #maxBuyVal#. Sellers have values that are randomly drawn between 0 and #maxSellVal#.

If you are a buyer, then you may purchase properties from sellers. If you manage to purchase all properties from all sellers then you will receive a package value for all the properties. Your profit will be the package value less the price that you paid for these properties. Only you will know your package value.

<Insert Treatment Specific Instructions Here>

You will now participate in a quick quiz. After completion of the quiz you will participate in #practiceRounds# practice rounds. You will have more time to make decisions in the practice rounds than during the actual experiment. In all #practiceRounds# practice rounds your earnings will not count towards your total payout.

Specific Instructions:
"buyer proposes sequential ultimatum"
First the buyer has #decisionTime# seconds to make a bid to seller 1. Then seller 1 has #decisionTime# seconds to decide whether to accept or reject this bid. If seller 1 accepts, then the buyer must pay this bid to seller 1. The buyer then has #decisionTime# seconds to make a bid to seller 2. Then seller 2 has #decisionTime# seconds to decide whether to accept or reject this offer. If seller 2 accepts, then the buyer must pay this bid to seller 2.

Example:
Assume the buyer has a package value of 81, seller 1 has a value of 26, and seller 2 has a value of 13. The buyer makes a bid of 31 to seller 1. If seller 1 accepts the bid, the buyer will pay 31 to seller 1 and seller 1 will receive a profit of 5 (31 – 26) from the contract. Only if seller 1 accepts the bid will the buyer be allowed to make a second bid to seller 2. Then the buyer makes a bid of 42 to seller 2. If seller 2 accepts the bid, the buyer will pay 42 to seller 2 and seller 2 will receive a profit of 29 (42 – 13). Only when both sellers accept the buyer's bids will the buyer receive any profit. In this case the buyer will receive a profit of 8 (81 – (31 + 42)).
Looking back at the previous example; if seller 1 accepted the buyer's bid but seller 2 declined it, then the buyer would pay seller 1 31 but receive a profit of -31 (0 – 31). The buyer must make a contract for both properties in order to realize his/her package value.

Sealed Optimal Split Specific Instructions
The buyer and each seller will have #decisionTime# seconds to simultaneously submit offers. Each subject will not see the offers placed by other subjects until all subjects have submitted offers. If the buyer's bid is equal to or greater than the sum of the sellers' asks then a contract will

be completed. If a contract is completed the buyer will pay his/her bid minus half the difference between the bid and the sum of the sellers' asks. Each seller will receive his/her ask plus a #profitFunction# of the difference between the bid and the sum of the sellers' asks.

For example, if there are two sellers and a single buyer and the buyer bids 80, seller 1 asks for 40 and seller 2 asks for 20, then a contract will be completed because 80 is greater than or equal to 60 (40+20). The difference between the buyer's bid and the sum of the sellers' asks is 20 (80-60). The buyer will pay 70 (80 - 20/2), seller 1 will receive 45 (40 + 20/4), and seller 2 will receive 25 (20 + 20/4).

If the buyer or any seller does not submit an offer or if the bid is less than the sum of asks then no contract will be completed.

"package offer" Specific Instructions
The buyer first has #decisionTime# seconds to submit a bid for all the sellers' properties. Each seller observes this bid. Then each seller has #decisionTime# seconds to submit an ask. If the bid is greater than or equal to the sum of the sellers' asks then a contract will be completed. If a contract is completed then the buyer will pay the bid. Each seller will receive his/her ask plus #profitFunction# the difference between the buyer's bid and the sum of the sellers' asks.

For example, if there are two sellers and a single buyer and the buyer bids 80, seller 1 asks for 40 and seller 2 asks for 20, then a contract will be completed because 80 is greater than or equal to 60 (40+20). The difference between the buyer's bid and the sum of the sellers' asks is 20 (80-60). The buyer will pay 80, seller 1 will receive 50 (40+10), and seller 2 will receive 30 (20+10).

If the buyer or any seller does not submit an offer or if the bid is less than the sum of asks then no contract will be completed.

Triple Auction Specific Instructions
The buyer and the sellers may submit offers at any time. A contract will be completed if the buyer's bid is (greater than or) equal to the sum of the sellers' asks. Each new bid must be greater than or equal to the previous bid. Each new ask must be less than or equal to the previous ask.

The buyer's will observe the total asks by the other players. This will be the sum of the sellers' asks. If the buyer bids at least this amount then a contract will be completed and the buyer will pay the sum of the sellers' asks and each seller will receive his/her ask.

Each seller will observe the effective bid from the other players. This will be the buyer's bid minus the sum of the other sellers' asks. If the seller asks for this value or lower, then a contract will be completed and the buyer will pay his bid and each seller will receive his/her ask.

Bargaining ends without a contract if no contract is made within #decisionTime# seconds.

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