An Experimental Study of Auctions with a Buy Price Under Private and Common Values*

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Abstract

eBay’s Buy It Now format allows a seller to list an auction with a “buy price” at which a bidder may purchase the item immediately and end the auction. When values are common, there is theoretically no revenue advantage to offering a buy price whether bidders are risk neutral or risk averse, while when values are private a buy price can be advantageous for the seller when bidders are risk averse. We report the results of laboratory experiments designed to determine whether in practice a buy price is advantageous to the seller. We find that a suitably chosen buy price yields a substantial increase in seller revenue when values are private, and a small (but statistically insignificant) increase in revenue when values are common. In both cases a buy price reduces the variance of seller revenue. A behavioral model which incorporates the winner’s curse and the overweighting by bidders of their own signal explains the common value auction data better than the rational model.

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1 Introduction

In a “buy it now” auction the seller sets a fixed price, termed a “buy price,” at which a bidder may purchase the item, thereby ending the auction immediately. If no bidder accepts the buy price, then in the ascending bid auction that follows, the bidder with the highest bid wins and pays the second highest bid. The buy-now auction format has proven to be extremely popular. Yahoo introduced the buy-now auction format in 1999. eBay followed with its own buy-now auction format in 2000 and by the end of 2001 about 40% of all eBay auctions were buy-now auctions (see Hof (2001)).

Several theoretical explanations for the popularity of buy prices have been proposed for private-value auctions. Reynolds and Wooders (2003) show for both eBay and Yahoo auctions that when bidders are risk averse then a suitably chosen buy price raises seller revenue; in this case the buy price extracts a risk premium from bidders who wish to avoid uncertainty over whether they win and the price that they pay. For eBay auctions, Mathews (2003) establishes that a seller can increase his revenue by setting a buy price when bidders are impatient; in this case the buy price extracts a premium from bidders who end the auction early. Mathews and Katzman (2006) establish that a buy price may be advantageous for a risk-averse seller as it reduces the variance of seller revenue.

In the present paper we investigate experimentally the properties of eBay buy-now auctions in both pure private value and pure common value settings. For private value auctions our objective is to determine whether a buy price raises seller revenue and whether it reduces the variance of revenue, as theory suggests it can. We find that in private value auctions the use of a buy price has a positive and statistically significant effect on seller revenue. We also find that a buy price lowers the standard deviation of revenue. In fact, the empirical c.d.f. of revenue in buy price auctions second order stochastically dominates revenue in auctions without a buy price, suggesting that a risk averse seller is better off with a buy price. These experimental results provide support to the main theoretical explanations offered for the use of buy prices in private value auctions.1

In private value auctions a bidder who accepts a buy price eliminates uncertainty about his payment (and hence his payoff). In common-value auctions, in contrast, a bidder who accepts a buy price eliminates uncertainty about his payment but not

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1 Since there is no meaningful delay in laboratory experiments, our results do not provide insight into whether a buy price can be used to exploit bidder impatience.
about his payoff, since he is also uncertain about the item’s value. Common value buy price auctions have, as a result, quite different theoretical properties than private value auctions, as shown in Shahriar (2008). In the common value setting we study, (i) any buy price that in equilibrium is accepted with positive probability reduces seller revenue when bidders are either risk neutral or risk averse, and (ii) seller revenue and the probability the buy price is accepted are both decreasing in the degree of bidder risk aversion. These different theoretical properties suggest that buy prices are worth studying experimentally in common value settings. Common value auctions are of practical interest as well since in some eBay auctions a common value model may be more appropriate than a private value model. Bajari and Hortacsu (2003), for example, argue that a common value model is correct for eBay coin auctions.2

Prior experimental studies of common value auctions, beginning with the seminal work of Kagel and Levin (1986), have found that bidders are naive, conditioning only on their own signal (and failing to condition on their rivals’ signals) when forming their bid.3 Buy-now auctions are of special interest since they provide a new mechanism to distinguish naive from rational bidding when values are common. To see this, suppose that bidders follow “cutoff” strategies, with a bidder accepting the buy price if his signal exceeds some cutoff “c,” and rejecting it otherwise. When all other bidders reject the buy price, this is informative to a naive bidder since he then infers that all his rivals have signals less than the cutoff c. A naive bidder, consequently, drops out earlier when the buy price is rejected than he would in an ascending bid auction where no buy price is offered. In contrast, a rational bidder forms his bid in an ascending bid auction by conditioning on both his own signal and on the highest signal of a rival bidder being equal to his own signal (see Milgrom and Weber (1982)). Hence a rational bidder who rejects the buy price (since his signal is less than c) obtains no information relevant to his bidding when all other bidders reject the buy price. He, therefore, chooses the same dropout price in an ascending bid auction, whether or not the auction has a buy price.

We find, in fact, that bidders drop out earlier when the buy price is rejected than they do in identical auctions where no buy price was offered. This finding provides additional support for the naive bidding model. We find that in the ascending bid

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2They find the sale price is decreasing in the number of bidders, which is consistent with common values.
3Kagel and Levin’s 1986 paper and their other foundational contributions on common-value auctions are conveniently assembled in their book Common-Value Auctions and the Winner’s Curse.
phase of the auction, bidders tend to overbid relative to equilibrium when they have low signals and underbid when they have high signals. This last finding is consistent with prior studies of ascending bid common-value auctions without a buy price. In contrast to the predictions of the rational model, (i) the use of a buy price in common value auctions had a small positive (although statistically insignificant) effect on seller revenue, and (ii) the buy price is accepted with high frequency, although theoretically it is always rejected if bidders are risk neutral or risk averse. We also find the variance of seller revenue is significantly lower in the buy-now auctions.

In order to better explain the common value data, we build on the naive bidding model of Kagel and Levin (1986) to develop and estimate a behavioral model of common value buy-now auctions. In our model a bidder fails to condition his value for the item on winning, both when deciding whether to accept the buy price and when deciding whether to drop out in the ascending bid phase of the auction (i.e., a bidder suffers from the "winner's curse" when making either type of decision). However, a bidder does update regarding his rivals' signals if none takes the buy price. The model also allows bidders to overweight their private information, and we find that overweighting of own signal is important in explaining the high frequency with which the buy price is accepted. The behavioral model explains (i) the high rate at which the buy price is accepted, (ii) the high than expected revenue realized in the buy-now auction, and (iii) that bidding behavior is less aggressive following the rejection of the buy price.

Results From Field Data

There are several important empirical studies of buy-now auctions based on field data from eBay. We focus here on their findings regarding which sellers tend to use a buy price and the revenue effects of a buy price. In an extensive study of Palm Vx auctions, Anderson, Friedman, Milam, and Singh (2004) find that sellers are more likely to use the buy-now auction format as they are more experienced. However, since their sample consists entirely of auctions which end with a sale, the effect on seller revenue of employing a buy price is not clear. Durham, Roelofs, and Standifird (2004) study a sample of 138 auctions of American silver dollars ending with a sale. They find that the 41 auctions listed with a buy price had an average selling price of $10.27, while the remaining auctions had an average selling price of $9.56, a statistically significant difference. They also find that a buy price tended to be offered by the more experienced sellers. Durham, Roelofs, and Standifird report, in
addition, the results of a field experiment in which they conducted 84 eBay auctions of 2001 American Eagle Silver dollars, with varying buy prices but the same low reserve price of $1.00. All of these auctions ended with a sale. The average sale price in auctions without a buy price was $8.82. Setting a buy price equal to $8.80 raised average revenue to $9.83, a difference which was statistically significant.4

Laboratory experiments complement the analysis of field data obtained from either naturally occurring or experimental auctions for several reasons. First, the theory doesn’t make unambiguous predictions regarding the revenue effect of buy prices when sellers are risk averse since, as shown in Mathews and Katzman (2006), a risk averse seller has an incentive to set a buy price even if it reduces expected revenue. Second, as noted earlier, the theoretical properties of a buy price differ depending on whether values are private or common. In the field it may be difficult to determine whether either pure private values or pure common values is appropriate, whereas in the lab the experimenter controls the structure of values. Third, in the field bidders may be both risk averse and impatient. The short duration of experiments in the lab allows the experimenter to focus on the effects of bidder risk aversion in isolation. Fourth, when analyzing field data one must control for the seller’s reputation, which is known to have an effect on price (see, e.g., Houser and Wooders (2006)), whereas reputation is not a factor in lab experiments. Finally, in a laboratory experiment one can focus on the effect of a buy price by varying whether or not the auction has a buy price, while holding all the other aspects of the auction – e.g., the number of bidders, the reserve price – fixed.

The present paper complements a number of other experimental studies of Internet auction formats. Ariely, Ockenfels, and Roth (2005), Ockenfels and Roth (2006), and Houser and Wooders (2005) study the effects of the auction closing rule, “hard” or “soft,” on bidding behavior. Ely and Hossain (2006) and Grey and Reiley (2007) use field experiments to investigate the reasons for, and consequences of, late bidding (or “sniping”) in hard close auctions. Salmon and Wilson (2008) is an experimental study of eBay’s “second chance offer” auction format.

The present paper is organized as follows. In Section 2 we present the experimental

4Durham, Roedofs, and Standifird remark that this difference is largely driven by several outliers in the buy-now auction where the sale price was unusually high. Since the $8.80 buy-price was accepted in only 1 of the 21 buy-now auctions, the revenue difference must be the result of more aggressive bidding when a buy price was originally offered (but disappeared following a bid) than when no buy price was offered at all.
design. The theoretical background on buy-now auctions is presented in Section 3. Section 4 presents our experimental results for both the private value and common value auctions. In Section 5 we estimate a behavioral model of common value buy-now auctions. Section 6 provides concluding remarks.

2 Experimental Design

We conducted experiments for the four treatments shown in Table 1. In the private value (henceforth PV) treatments, each bidder’s value for the item was an independent draw from the $U[0, 10]$ distribution. In the common value (henceforth CV) treatments each bidder’s signal was an independent draw from the $U[0, 10]$ distribution; the value of the item was the same for each bidder and equal to the average of the signals.\(^5\) For expositional convenience we used the same distribution for value/signal draws in both the private and common value treatments. This implies, however, no theoretical connection between the two treatments.

We conducted both ascending bid auctions and buy-now auctions. In the ascending bid auctions the price increased by $.25 per second so long as at least one bidder remained active. At any point a bidder could exit the auction by clicking on a “Drop Out” button. Bidders did not observe the number of bidders remaining in the auction, i.e., they did not observe when a rival bidder dropped out. The auction ended when only one bidder remained; the remaining bidder won the auction and paid a price equal to the amount at which the last bidder dropped out.\(^6\) At the end of each round, each bidder observed the sale price. In the common value auctions bidders observed, in addition, the signals of all four bidders. There was no reserve price, and the clock began ascending from a bid of $0.

The buy-now auctions had two stages. At the first stage the four bidders simultaneously decided whether to accept or reject the buy price. The buy price was $8.10 in the private value auctions and was $5.60 in the common value auctions. (We motivate these choices in the next section.) If a bidder accepted the buy price, then he won the item at the buy price and the auction ended.\(^7\) If all the bidders rejected the

\(^5\)Experimental papers in which the common value is the average of the bidders’ signals include Avery and Kagel (1997), Holt and Sherman (2000), Goeree and Offerman (2002).

\(^6\)This auction format is sometimes referred to as a Japanese, or button auction.

\(^7\)If more than one bidder accepted the buy price, then the item was randomly allocated to one of the accepting bidders.
buy price, then at the second stage the item was sold via the ascending bid auction described above.

As eBay implements buy-now auctions, the buy-price disappears as soon as the first bid is placed and hence it is only available to the first bidder in the auction.\(^8\) Our experimental design, in contrast, follows the developed theory (Mathews and Katzman (2006), Reynolds and Wooders (2003), Shahriar (2008)) which models bidders as simultaneously deciding whether to accept or reject the buy price. This modeling choice, however, is not significant: If only the first bidder to arrive can accept the buy price, the main theoretical results are qualitatively the same, e.g., when bidders are risk averse a buy price may raise seller revenue when values are private but it can not raise revenue when values are common.

The experiments were conducted at the University of Arizona where subjects were recruited in groups of eight. Each group of 8 subjects was split into two groups of four bidders, and each group of four bidders participated in 30 periods of an auction.\(^9\) We refer to a single group of four bidders participating in 30 rounds of a given auction format as a “session.” We conducted six sessions for each of the four treatments, and hence a total of 96 subjects participated in the experiments. The bidders’ values/signals were determined randomly once, i.e., the same set of 120 values/signals (4 bidders per auction and 30 auctions) was used in all 24 sessions.\(^10\) Table 1 summarizes our experimental design.

<table>
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<tr>
<th></th>
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<th>Dist. of Values/Signals</th>
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<td>30</td>
<td>4</td>
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<tr>
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<td>$8.10</td>
<td>6</td>
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<td>U[$0, $10]</td>
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<tr>
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<td>30</td>
<td>4</td>
</tr>
<tr>
<td>Buy-now</td>
<td>$5.60</td>
<td>6</td>
<td>30</td>
<td>4</td>
<td>U[$0, $10]</td>
</tr>
</tbody>
</table>

Table 1 - Experimental Design

\(^8\)eBay has recently begun experimenting with Yahoo!-style buy prices which are available to all bidders throughout the entire auction.

\(^9\)The composition of each group was fixed, but to minimize the potential for repeated game effects the subjects were not informed of this fact.

\(^10\)We use the same values/signals in each session so that differences in outcomes are due to the different auction mechanisms (and the idiosyncratic differences in bidders) rather than different realizations of values.
In common value auctions it has been observed in prior experiments (e.g., Kagel and Levin (1986) and Kagel, Levin, Battalio and Meyer (1989)) that subjects sometimes go “bankrupt,” with their accumulated earnings becoming negative. Bankruptcy affects a bidder’s incentives as he is no longer liable for his losses. A low but positive balance also affects a bidder’s incentives as he can lose at most his current balance if he wins an auction and the price exceeds the item’s value. In our common value experiments, a bidder began with an initial balance of $25 (experimental dollars) and was declared bankrupt if his current balance fell below $5.\textsuperscript{11} In the private value sessions, each subject began with an initial balance of $5 and was declared bankrupt if his balance fell below $0. A bankrupt bidder would exit the experiment and be replaced by an additional subject who was standing by. In fact, no subject went bankrupt over the course of the experiment.\textsuperscript{12}

3 Theoretical Background

The theoretical foundation of our experimental design is Reynolds and Wooders (2003) and Shahriar (2008) for, respectively, private and common value buy-now auctions. In each case, there are $n$ bidders who are assumed to have constant absolute risk aversion (CARA) utility function $u(w) = (1 - e^{-\alpha w})/\alpha$, where $\alpha \geq 0$ is the index of risk aversion. Since $\lim_{\alpha \to 0} u(w) = w$, then $\alpha = 0$ corresponds to risk neutrality. Let $B$ denote the buy price in a buy-now auction. Denote by $F(v)$ the cumulative distribution function of values/signals with support $[\underline{v}, \bar{v}]$. Let $G(v) = F(v)^{n-1}$ be the c.d.f. of the highest of $n-1$ values/signals. The densities of $F$ and $G$ are denoted by $f$ and $g$, respectively. For our experimental design we have $n = 4$ and $F(v) = \frac{v}{10}$ for $v \in [0, 10]$. To simplify, we assume the seller sets no reserve price.

\textsuperscript{11}The $5$ amount was chosen as it is the most a bidder can lose as a result of winning an auction (when his rivals follow the symmetric equilibrium). With risk-neutral bidders, the highest equilibrium drop out price is $7.50$ (for a bidder whose signal is $10.00$) and hence the winning bidder pays at most $7.50$. In this case, the value of the item is at least $(10.00 + 0 + 0 + 0)/4 = 2.50$ and hence a winning bidder loses at most $5.00$.

\textsuperscript{12}In the common value auctions each experimental dollar was converted into $.65. Subjects were paid an average of $15.32$ and $16.35$, respectively, in auctions with and without a buy price. In the private value auctions, at the end of the experiment, each experimental dollar was converted into $.80. Subjects were paid an average of $13.62$ and $16.09$, respectively, in auctions with and without a buy price. Each session lasted between 30 and 40 minutes.
PRIVATE VALUE AUCTIONS

In an ascending bid private-value auction it is a dominant strategy for a bidder to drop out of the auction when the bid reaches his value. Equilibrium in a private-value buy price auction is characterized by a cutoff $c$ such that a bidder accepts the buy price if his value exceeds $c$ and rejects it otherwise. Suppose that a bidder’s value is $v$ and all his rivals employ the same cutoff $c$. His payoff to accepting the buy price is

$$U^A_\alpha(v, c) = u(v - B)Q(c),$$

where $Q(c)$ is the probability that the bidder wins the auction. If the bidder rejects the buy price, then he wins the auction only if he has the highest value and all his rivals also reject the buy price. His expected payoff is

$$U^R_\alpha(v, c) = \int_{\min\{v, c\}}^{\infty} u(v - y)dG(y).$$

A cutoff $c^*$ is a symmetric Bayes Nash equilibrium if $U^R_\alpha(v, c^*) > U^A_\alpha(v, c^*)$ for all $v \in [\bar{v}, c^*)$ and $U^R_\alpha(v, c^*) < U^A_\alpha(v, c^*)$ for all $v \in (c^*, \tilde{v}]$, i.e., it is optimal for a bidder to accept the buy price if his value exceeds $c^*$ and it’s optimal for him to reject the buy price if his value is less than $c^*$.\(^\text{13}\)

Figure 1 shows seller revenue as a function of the buy price when $\alpha = 0$ (i.e., bidders are risk neutral) and, to illustrate the effect of bidder risk aversion, when $\alpha = 1$. When bidders are risk neutral then, as Figure 1 shows, any buy price accepted in equilibrium with positive probability (i.e., with $B < $7.50) reduces the seller’s expected revenue.\(^\text{15}\) A buy price above $7.50 is not accepted in equilibrium and hence the seller obtains the expected revenue of the ascending bid auction ($$6.00). In contrast, when $\alpha = 1$ then there is a range of buy prices which raises seller revenue. If, for example, the buy price is $8.10 (as in our experiment), then the equilibrium cutoff is $9.11, the buy price is accepted by at least one bidder with probability 0.31, and

\(^\text{13}\)Assuming the item is allocated randomly among the bidders who accept, the probability of winning if exactly $l$ other bidders accept is $\frac{1}{l+1}$, and hence

$$Q(c) = \sum_{l=0}^{n-1} \binom{n-1}{l} \frac{1}{l+1} (1 - F(c))^l F(c)^{n-1-l}.$$

\(^\text{14}\)Reynolds and Wooders (2003) establish that when bidders are CARA risk averse then there is a unique symmetric equilibrium in cutoff strategies.

\(^\text{15}\)This is a consequence of the fact than an ascending bid auction without a reserve is an optimal selling mechanism within the class of efficient mechanisms, when bidders are risk neutral.
the seller’s expected revenue is $6.29.\textsuperscript{16} A buy price which exceeds $8.60 is rejected by all bidders, and again yields the expected revenue of the ascending bid auction.

Figure 1 goes here.

We chose a buy price of $8.10 since it will (theoretically) raise seller revenue for even moderate degrees of risk aversion.

**Common Value Auctions**

Let $x_i$ denote the signal of bidder $i$. In an ascending bid common value auction without a buy price, the symmetric equilibrium bidding function when bidders have index of risk aversion $\alpha$, denoted by $b_\alpha(x)$, satisfies

$$E[u(v - b_\alpha(x))|x_1 = x, z = x] = 0,$$

where $v = (x_1 + \cdots + x_n)/n$ is the true value of the item and $z = \max\{x_2, \ldots, x_n\}$ is the highest signal of a rival bidder.\textsuperscript{17} (See Milgrom and Weber (1982) and Levin and Harstad (1986).) When bidders have CARA then

$$b_\alpha(x) = \frac{1}{\alpha} \ln \left[ \frac{1}{E[e^{-\alpha v}|x_1 = x, z = x]} \right],$$

which reduces to $b_0(x) = \frac{n+2}{2n} x$ when bidders are risk neutral. In common value auctions bidders drop out earlier as they become more risk averse, with $b_\alpha(x) < b_0(x)$ for $\alpha > 0$ and $x > v$.

We characterize equilibrium in a common-value buy price auction by a cutoff $c$ such that a bidder accepts the buy price if his signal exceeds $c$ and rejects it otherwise. Suppose that a bidder’s signal is $x$ and all his rivals employ the same cutoff $c$. Then his payoff to accepting the buy price is

$$\hat{U}_\alpha^A(x, c) = \sum_{l=0}^{n-1} \left[ \binom{n-1}{l} F(c)^{n-1-l}(1 - F(c))^l \frac{1}{l+1} u_l \right],$$

where $u_l$ is the expected utility to a bidder of winning the item at price $B$ when his own signal is $x$, $l$ rival bidders have signals above $c$, and $n - 1 - l$ rivals have signals

\textsuperscript{16}Any buy price above $7.50 will raise seller revenue provided the bidders are sufficiently risk averse that it is accepted with positive probability.

\textsuperscript{17}For notational convenience, we adopt the perspective of bidder 1. Since bidders do not observe the drop out prices of their rivals, the auction is strategically equivalent to a second-price sealed-bid auction.
below $c$.\footnote{In particular}

If all the bidders reject the buy price, then the equilibrium bidding function in the ascending bid auction that follows continues to be given by \eqref{U0}. In other words, equilibrium dropout prices are the same in an ascending bid auction without a buy price and an ascending bid auction reached following the rejection of a buy price. Intuitively this is because a bidder drops out at the bid where he is indifferent between winning or losing the auction, conditional on the highest signal of a rival ($z$) being equal to his own signal. A bidder who rejects the buy price (since his signal $x$ is less than $c$) and who observes that all his rivals also reject the buy price, can infer that the highest signal of a rival is also less than $c$ (i.e., $z < c$). Since he conditions his dropout price on $z = x$ (with $x$ less than $c$), then observing $z < c$ is uninformative.

Consider a bidder with signal $x$ who rejects the buy price. He wins the ascending bid auction and pays $b_\alpha(z)$ if he has the highest signal (i.e., $z < x$) and no other bidder accepts the buy price (i.e., $z < c$). Hence, his payoff to rejecting the buy price is

$$
\hat{U}_\alpha^R(x, c) = \int_{\underline{v}}^{\min\{x, c\}} \left[ \int_{\underline{v}}^{z} \cdots \int_{\underline{v}}^{z} u \left( \frac{x + z + \sum_{i=2}^{n} x_i}{n} - b_\alpha(z) \right) \frac{f(x_3)}{F(z)} dx_3 \cdots \frac{f(x_n)}{F(z)} dx_n \right] g(z) dz.
$$

A cutoff $c^*$ is a \textbf{symmetric Bayes Nash equilibrium} if $\hat{U}_\alpha^R(x, c^*) > \hat{U}_\alpha^A(x, c^*)$ for all $x \in [\underline{v}, c^*)$ and $\hat{U}_\alpha^R(x, c^*) < \hat{U}_\alpha^A(x, c^*)$ for all $x \in (c^*, \overline{v}]$.\footnote{Shahriar (2008) establishes that a symmetric BNE exists in common value buy now auctions under general conditions when bidders have CARA.}

Figure 2 illustrates the equilibrium cutoff for our experimental design, with a buy price of $5.60$, when $\alpha = 0$ and when $\alpha = 1$.\footnote{Numerical calculations establish that there is only one symmetric equilibrium in cutoff strategies in both cases.} When $\alpha = 0$, the equilibrium cutoff is $c = 9.96$. In particular, a bidder with a signal $x > 9.96$ obtains a higher payoff accepting the buy price than rejecting it (since $\hat{U}_0^R(x, 9.96) < \hat{U}_0^A(x, 9.96)$), when all his rivals employ a cutoff of $9.96$. A bidder with a signal $x < 9.96$ obtains a higher payoff rejecting the buy price. If $\alpha = 1$, the equilibrium cutoff is $c^* = 10$ and the buy price is rejected by all bidders (since $\hat{U}_1^R(x, 10) > \hat{U}_1^A(x, 10)$ for all $x \leq 10$).

Figure 2 goes here.

\begin{align*}
\int_{\underline{v}}^{\overline{v}} \cdots \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} \int_{\underline{v}}^{\overline{v}} u \left( \frac{x + \sum_{i=1}^{n} x_i}{n} - B \right) \frac{f(x_2)}{F(c)} dx_2 \cdots \frac{f(x_{n-1})}{F(c)} dx_{n-1} \frac{f(x_n)}{1 - F(c)} dx_n.
\end{align*}
As illustrated in Figure 2, when values are common then bidders are less likely to accept the buy price when risk averse. In our experiment, no bidder had a signal above $9.96 and hence, according to the (rational) theory, no risk neutral or risk averse bidder will accept the buy price.

Figure 3 shows seller revenue as a function of the buy price for the cases \( \alpha = 0 \) and, for illustration, when \( \alpha = 1 \). If \( \alpha = 0 \) then any buy price above $5.63 is rejected and the seller obtains the expected revenue of the ascending bid auction ($4.50). If \( \alpha = 1 \) then any buy price above $4.98 is rejected. In both cases, the introduction of a buy price reduces seller revenue if the buy price is accepted in equilibrium with positive probability. Furthermore, seller revenue is lower when bidders are risk averse than when they are risk neutral.

Figure 3 goes here.

Prior research has shown that bidders suffer from the winner’s curse in common value auctions. We chose a buy price of $5.60 since it offers the possibility of raising seller revenue if subjects also suffer from the winner’s curse when choosing whether to accept the buy price.

The preceding analysis shows that risk aversion has dramatically different effects in private and common value buy-now auctions. In private value auctions, seller revenue and the probability the buy price is accepted both increase as bidders become more risk averse. In common value auctions an increase in risk aversion has the opposite effect.

4 Results

Private Value Auctions

The left panel of Table 2 shows the mean per-period seller revenue, the standard deviation of revenue, and auction efficiency in each of the six private value buy-now sessions. We measure auction efficiency as the fraction of auctions in which the highest bidder wins. The buy price was accepted in 81 of the 180 auctions. The right panel of Table 2 shows the same data for the six sessions of private value ascending bid auctions.
Buy-now, $B = $8.10

<table>
<thead>
<tr>
<th>Session</th>
<th>Rev.</th>
<th>s.d. Rev.</th>
<th>Efficiency</th>
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<td>1</td>
<td>$6.33</td>
<td>$1.84</td>
<td>86.67%</td>
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<td>93.33%</td>
</tr>
<tr>
<td>5</td>
<td>$6.25</td>
<td>$1.82</td>
<td>80.00%</td>
</tr>
<tr>
<td>6</td>
<td>$6.72</td>
<td>$1.88</td>
<td>86.67%</td>
</tr>
<tr>
<td>Mean</td>
<td>$6.47</td>
<td>$1.87</td>
<td>89.44%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Session</th>
<th>Rev.</th>
<th>s.d. Rev.</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>$6.20</td>
<td>$1.97</td>
<td>100.00%</td>
</tr>
<tr>
<td>8</td>
<td>$6.04</td>
<td>$2.06</td>
<td>93.33%</td>
</tr>
<tr>
<td>9</td>
<td>$5.98</td>
<td>$2.00</td>
<td>90.00%</td>
</tr>
<tr>
<td>10</td>
<td>$6.17</td>
<td>$2.08</td>
<td>96.67%</td>
</tr>
<tr>
<td>11</td>
<td>$6.11</td>
<td>$2.08</td>
<td>86.67%</td>
</tr>
<tr>
<td>12</td>
<td>$5.86</td>
<td>$2.25</td>
<td>93.33%</td>
</tr>
<tr>
<td>Mean</td>
<td>$6.06</td>
<td>$2.07</td>
<td>93.33%</td>
</tr>
</tbody>
</table>

Table 2: Revenue and Efficiency With and Without a Buy Price

Averaging across all 6 sessions, seller revenue was $6.06 in the ascending bid auctions, which is less than the mean revenue of $6.47 in the buy-now auctions.\(^{21}\) The lowest revenue achieved in a session of buy-now auctions ($6.25) exceeded the highest revenue achieved in any ascending bid auction session ($6.20). Applying the Mann-Whitney U test to the two samples, we can reject at the 1% level the hypothesis that the revenues of each auction format are drawn from the same distribution ($U = 0$ with a $p$-value of .002).\(^{22}\) Hence a risk-neutral seller is better off with a buy price. Table 2 also shows that the standard deviation of revenue is lower in the buy-now auctions, a difference that is significant at the 1% level ($U = 0$ with a $p$-value of .002).

Further insight into the effect of introducing a buy price is revealed by Figure 4, which shows the empirical \(c.d.f.\) of revenue in the 180 buy-now auctions and the 180 ascending bid auctions.

Figure 4 goes here.

A buy price reduces the likelihood that seller revenue is either very low or very high. Introducing a buy price of $8.10 reduces by about 10% the chance that the seller will obtain less than $6.00. It reduces by about 25% the chance that the seller will obtain more than $8.10.\(^{23}\) The empirical \(c.d.f.\) of revenue in a buy price auction

\(^{21}\) Conditional on the realized values, mean theoretical revenue in the ascending auction is $6.19, which is slightly more than the $6.06 actually obtained.

\(^{22}\) See Siegel (1956), pp. 116-121.

\(^{23}\) In a buy-now auction it is theoretically possible for seller revenue to exceed $B$. If both the highest and second highest values are below the equilibrium cutoff but above $B$, then all bidders reject the buy price and the price in the ascending bid auction is the second highest value.
(nearly) second-order stochastically dominates the c.d.f. of revenue in an ascending bid auction, which suggests that a risk averse seller is also better off with a buy price.

Theoretically, the ascending bid auction is 100% efficient since the bidder with the highest value wins the auction. The introduction of a buy price, however, reduces auction efficiency since when more than one bidder accepts the buy price then the item is allocated randomly to one of these bidders. Table 2 shows the percentage of auctions in which the bidder with the highest value won. It shows that the introduction of a buy price leads to only to a modest reduction in auction efficiency (from 93.33% to 89.44%). Applying the Mann-Whitney U test we cannot reject the hypothesis that the efficiency of the two auction formats is the same ($U = 11.5$ and $p = .394$). An alternative measure of efficiency is the ratio of realized surplus to the maximum total surplus. By this measure, both the ascending bid and the buy-now auction are highly efficient, realizing over 99% of the theoretical total surplus.

Table 3(a) compares the buy-now auctions in which the buy price was accepted to identical ascending bid auctions, i.e., to ascending bid auctions where the bidders’ values are the same. Consider, for example, the first row of Table 3(a). The buy price was accepted in 12 of the Session 1 auctions and in each of these auctions seller revenue was $8.10. Average revenues in the 12 identical auctions in the six ascending bid auction sessions were $7.57, $7.49, $7.39, $7.68, $7.55, and $7.54. Averaging revenues across the 6 ascending bid sessions yields a revenue of $7.54. The table shows that in each of the 36 possible pairwise comparisons, seller revenue is higher when the buy price is accepted than in the identical ascending bid auctions.

<table>
<thead>
<tr>
<th>Buy Now – B Accepted</th>
<th>Identical Ascending Bid Auction Sessions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1 (12 auctions)</td>
<td>$8.10</td>
<td>$7.57</td>
<td>$7.49</td>
<td>$7.39</td>
<td>$7.68</td>
<td>$7.55</td>
<td>$7.54</td>
<td>$7.54</td>
</tr>
<tr>
<td>Session 2 (9 auctions)</td>
<td>$8.10</td>
<td>$6.99</td>
<td>$6.89</td>
<td>$6.88</td>
<td>$7.01</td>
<td>$6.86</td>
<td>$6.58</td>
<td>$6.87</td>
</tr>
<tr>
<td>Session 3 (14 auctions)</td>
<td>$8.10</td>
<td>$7.19</td>
<td>$7.02</td>
<td>$7.00</td>
<td>$7.23</td>
<td>$7.05</td>
<td>$7.05</td>
<td>$7.09</td>
</tr>
<tr>
<td>Session 4 (16 auctions)</td>
<td>$8.10</td>
<td>$7.36</td>
<td>$7.29</td>
<td>$7.23</td>
<td>$7.43</td>
<td>$7.35</td>
<td>$7.14</td>
<td>$7.30</td>
</tr>
<tr>
<td>Session 5 (12 auctions)</td>
<td>$8.10</td>
<td>$7.74</td>
<td>$7.66</td>
<td>$7.55</td>
<td>$7.83</td>
<td>$7.70</td>
<td>$7.66</td>
<td>$7.69</td>
</tr>
<tr>
<td>Session 6 (18 auctions)</td>
<td>$8.10</td>
<td>$7.09</td>
<td>$7.02</td>
<td>$6.96</td>
<td>$7.16</td>
<td>$7.03</td>
<td>$6.91</td>
<td>$7.03</td>
</tr>
</tbody>
</table>

Table 3(a): Seller Revenue Conditional on Acceptance of the Buy Price

Conditional on the buy price being accepted, seller revenue ($8.10) is $0.85 more (a 12% increase) than what he obtains on average ($7.25 across the six sessions) without a buy price.
Table 3(b) compares buy-now auctions in which the buy price was rejected to the identical ascending bid auctions. The buy price was rejected in 18 of the Session 1 auctions and in these auctions mean seller revenue was $5.16. Averaging revenues across the 6 ascending bid sessions yields a revenue of $5.07, a difference of only $.09.

<table>
<thead>
<tr>
<th>Buy Now – B rejected</th>
<th>Identical Ascending Bid Auction Sessions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Session 1 (18 auctions)</td>
<td>$5.16</td>
</tr>
<tr>
<td>Session 2 (21 auctions)</td>
<td>$5.81</td>
</tr>
<tr>
<td>Session 3 (16 auctions)</td>
<td>$5.19</td>
</tr>
<tr>
<td>Session 4 (14 auctions)</td>
<td>$4.67</td>
</tr>
<tr>
<td>Session 5 (18 auctions)</td>
<td>$5.01</td>
</tr>
<tr>
<td>Session 6 (12 auctions)</td>
<td>$4.65</td>
</tr>
</tbody>
</table>

Table 3(b): Seller Revenue Conditional on Rejection of the Buy Price

In fact, the realized revenue in each buy-now auction session is generally within a few cents of the average revenue obtained across sessions of the identical ascending bid auctions. This suggests there is no difference in bidding behavior between when the buy price is offered and rejected, and when there is no buy price.

To determine whether bidders follow their dominant strategy of dropping out when the bid reaches their value, we estimated a censored regression linear bidding model. In each auction we observe the dropout prices of three bidders, but not the dropout price of the bidder who wins the auction. Assume that the dropout price of the \( i \)-th bidder, \( i \in \{1, 2, 3, 4\} \), in auction \( t \) is given by

\[
b^t_i = \gamma + \beta v^t_i + \varepsilon^t_i,
\]

where \( v^t_i \) is the value of bidder \( i \) in auction \( t \), and \( \varepsilon^t_i \) is distributed according to \( N(0, \sigma^2) \). For auction \( t \), let \( k^t \in \{1, 2, 3, 4\} \) denote the winner, i.e., bidder \( k^t \) has the highest (but unobserved) dropout price, and let \( \bar{b}^t \) be the highest observed dropout price. The censored regression likelihood function is given by

\[
L = \frac{1}{T} \sum_{t=1}^{T} \ln \left[ 1 - \Phi \left( \frac{\bar{b}^t - \gamma - \beta v^t_{k^t}}{\sigma^2} \right) \right] \prod_{i \in \{1,2,3,4\}\setminus\{k^t\}} \frac{1}{\sigma^2} \phi \left( \frac{b^t_i - \gamma - \beta v^t_i}{\sigma^2} \right),
\]
where $\Phi$ and $\phi$ are, respectively, the c.d.f. and p.d.f. of the standard normal, and where $T$ is the number of auctions. According to the theory, the intercept is $\gamma = 0$ and the slope is $\beta = 1$.

Table 4 reports the maximum likelihood estimates of $\gamma$ and $\beta$. The first column shows the results based on the 180 ascending bid auctions. While the intercept is not significantly different from zero, the slope is significantly less than one, and suggests that bidders drop out at approximately 95% of their value. The second column is based on the 99 auctions in which the buy price is rejected. In this case the intercept is significantly less than zero – bidders drop out at a price $\$11$ below their value – while the slope is not significantly different from one. According to the $z$-test, however, the coefficient estimates are not significantly different between the two data sets, which provides further evidence that bidding behavior does not differ following the rejection of the buy price. For both data sets, the departure from value bidding is small in economic terms.

<table>
<thead>
<tr>
<th></th>
<th>Ascending</th>
<th>Buy-now ($B$ rejected)</th>
<th>$z$-test ($p$-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\gamma}$</td>
<td>0.070 (0.074)</td>
<td>-0.115 (0.036)</td>
<td>0.18</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>0.947 (0.017)</td>
<td>1.003 (0.013)</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 4: ML Estimated Bidding Functions in PV auctions

To test whether the bidders’ decisions to accept or reject the buy price depend on their values, we run Probit regressions in which the binary dependent variable is 1 if the bidder accepts the buy price and is 0 otherwise. Table 5 below reports the results from two Probits, with the first using all the data and the second using only those observations in which the bidder’s value exceeded the buy price.\textsuperscript{24}

<table>
<thead>
<tr>
<th></th>
<th>All Data</th>
<th>$v_i \geq $8.10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Marginal</td>
</tr>
<tr>
<td>Constant</td>
<td>-10.260 (1.114)</td>
<td>0.00</td>
</tr>
<tr>
<td>Value $v_i$</td>
<td>1.161 (0.246)</td>
<td>3.09e-05 (6.31e-05)</td>
</tr>
</tbody>
</table>

Table 5: Probit – Decision to Accept Buy Price in PV auctions

\textsuperscript{24}We find no evidence that the decision to accept the buy price depends on experience. In particular, if one introduces a constant dummy for the last 15 rounds (in either the private value auctions here, or the common value auctions reported below), it is insignificantly different from zero.

16
In both cases, the probability that a bidder accepts the buy price increases significantly as his value increases.\textsuperscript{25}

The index of bidder risk aversion can be estimated from the bidders’ decisions to accept or reject the buy price.\textsuperscript{26} Let \( c^*_\alpha \) denote the equilibrium cutoff when bidders have index of risk aversion \( \alpha \), i.e., \( c^*_\alpha \) is the value of \( c \) solving \( U^A_\alpha(c, c) = U^R_\alpha(c, c) \). For a bidder with value \( v \), the difference in the payoff between accepting or rejecting the buy price is denoted by \( \Delta_\alpha(v) \), where

\[
\Delta_\alpha(v) = U^A_\alpha(v, c^*_\alpha) - U^R_\alpha(v, c^*_\alpha).
\]

Let \( D^t_i \) be a dummy variable indicating whether bidder \( i \) in auction \( t \) accepted the buy price, and let \( v^t_i \) be the value of bidder \( i \) in auction \( t \). Our econometric model is that

\[
D^t_i = \begin{cases} 
1 & \text{if } \Delta_\alpha(v^t_i) \geq \varepsilon^t_i \\
0 & \text{if } \Delta_\alpha(v^t_i) < \varepsilon^t_i,
\end{cases}
\]

where \( \varepsilon^t_i \) is \( N(0, \sigma_1) \). The likelihood function for the \( t^{th} \) buy-now auction is

\[
L^t = \begin{cases} 
1 - \prod_{i=1}^4 \left[ 1 - \Phi \left( \frac{\Delta_\alpha(v^t_i)}{\sigma_1} \right) \right] & \text{if } D^t_i = 1 \text{ for some } i \\
\prod_{i=1}^4 \left[ 1 - \Phi \left( \frac{\Delta_\alpha(v^t_i)}{\sigma_1} \right) \right] & \text{otherwise},
\end{cases}
\]

where \( \Phi \) and \( \phi \) are, respectively, the c.d.f. and p.d.f. of the standard normal. Maximizing \( \frac{1}{T} \sum_{t=1}^T L^t \) with respect to \( \alpha \) (and \( \sigma_1 \)), where \( T \) is the number of auctions, yields the estimate \( \hat{\alpha} = 1.092 \) (0.0003) and an equilibrium cutoff of \( c^*(\hat{\alpha}) = 9.028 \) (0.002). Standard errors are given in parentheses. Hence the bidders’ decisions whether to accept the buy price are consistent with risk aversion.

**Common Value Auctions**

The left panel of Table 6 shows average per-period seller revenue in the six common value buy-now auction sessions. The buy price was accepted in 142 of the 180 auctions. The right panel shows the average seller revenue in the six ascending auction

\textsuperscript{25}For values exceeding \$8.10 the mean is \$9.24. Hence the probit estimates suggest increasing the value of a bidder from \$9.24 to \$9.34, a \$.10 increase, increases the probability of accepting the buy price by .0266.

\textsuperscript{26}The bidders’ dropout prices do not contribute to identifying \( \alpha \) since regardless of the value of \( \alpha \), it is a dominant strategy for a bidder to drop out when the bid reaches his value.
sessions.

<table>
<thead>
<tr>
<th>Session</th>
<th>Rev.</th>
<th>s.d. Rev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5.05</td>
<td>$0.85</td>
</tr>
<tr>
<td>2</td>
<td>$5.46</td>
<td>$0.51</td>
</tr>
<tr>
<td>3</td>
<td>$5.23</td>
<td>$0.85</td>
</tr>
<tr>
<td>4</td>
<td>$5.43</td>
<td>$0.47</td>
</tr>
<tr>
<td>5</td>
<td>$5.35</td>
<td>$0.78</td>
</tr>
<tr>
<td>6</td>
<td>$5.00</td>
<td>$1.08</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Session</th>
<th>Rev.</th>
<th>s.d. Rev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>$4.84</td>
<td>$0.85</td>
</tr>
<tr>
<td>8</td>
<td>$4.64</td>
<td>$0.97</td>
</tr>
<tr>
<td>9</td>
<td>$5.11</td>
<td>$1.15</td>
</tr>
<tr>
<td>10</td>
<td>$5.37</td>
<td>$1.03</td>
</tr>
<tr>
<td>11</td>
<td>$4.81</td>
<td>$0.79</td>
</tr>
<tr>
<td>12</td>
<td>$5.49</td>
<td>$1.36</td>
</tr>
</tbody>
</table>

Table 6: Revenue With and Without a Buy Price

Introducing the $5.60 buy price raises revenue by an average of $.21 per auction, but this difference is insignificant according to the Mann-Whitney U test ($U = 12$ and $p = .394$). Both auctions yield more than the risk-neutral theoretical revenue of $4.50. The buy price does, however, reduce the standard deviation of seller revenue; we can reject at the 10% level that the standard deviation of revenue is the same in the two auction formats ($U = 7$ and $p = .094$). The reduction in the standard deviation of seller revenue is apparent in Figure 5, which shows the empirical c.d.f. of revenue in the 180 common value buy-now auctions and the 180 common-value ascending auctions.

Figure 5 goes here.

Table 7(a) compares revenue in auctions in which the buy price is accepted to revenue in the identical ascending bid auctions, and is constructed in the same fashion as Table 3(a). For each buy now auction session, revenue when the buy price is accepted ($5.60) exceeds average revenue across the identical ascending bid auction
sessions (show in the right most column of the table).

<table>
<thead>
<tr>
<th>Buy Now – B Accepted</th>
<th>Identical Ascending Bid Auction Sessions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Session 1 (17 auctions)</td>
<td>$5.60</td>
</tr>
<tr>
<td>Session 2 (23 auctions)</td>
<td>$5.60</td>
</tr>
<tr>
<td>Session 3 (25 auctions)</td>
<td>$5.60</td>
</tr>
<tr>
<td>Session 4 (26 auctions)</td>
<td>$5.60</td>
</tr>
<tr>
<td>Session 5 (27 auctions)</td>
<td>$5.60</td>
</tr>
<tr>
<td>Session 6 (20 auctions)</td>
<td>$5.60</td>
</tr>
</tbody>
</table>

Table 7(a): Seller Revenue Conditional on Acceptance of the Buy Price

The seller obtains $5.60 when the buy price is accepted, which is $0.42 higher than what he obtains on average in the identical ascending bid auctions.

Table 7(b) compares seller revenue in the two auction formats conditional on the buy price being rejected. As noted earlier, in the rational bidding model the price at which a bidder drops out in the ascending bid auction is theoretically the same when there is no buy price and when there was a buy price but it was rejected by all the bidders (when bidders’ values are the same in each case) – a bidder’s drop out price is given by (1). In the data, however, seller revenue is frequently lower following the rejection of the buy price. For example, the third row of Table 7(b) shows that the buy price was rejected in five of the Session 3 buy-now auctions and average seller revenue in these auctions was $3.39. This is less revenue than what was obtained in each of the six identical ascending bid auction sessions.

<table>
<thead>
<tr>
<th>Buy Now – B rejected</th>
<th>Identical Ascending Bid Auction Sessions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Session 1 (13 auctions)</td>
<td>$4.33</td>
</tr>
<tr>
<td>Session 2 (3 auctions)</td>
<td>$4.15</td>
</tr>
<tr>
<td>Session 3 (5 auctions)</td>
<td>$3.39</td>
</tr>
<tr>
<td>Session 4 (4 auctions)</td>
<td>$4.31</td>
</tr>
<tr>
<td>Session 5 (3 auctions)</td>
<td>$3.10</td>
</tr>
<tr>
<td>Session 6 (10 auctions)</td>
<td>$3.81</td>
</tr>
</tbody>
</table>

Table 7(b): Seller Revenue Conditional on Rejection of the Buy Price

We regressed revenue on a treatment dummy in a subset of the data consisting of all 180 ascending auctions and the 38 buy-now auctions where the buy price was
rejected. This regression shows that seller revenue decreases by $1.09 conditional on the rejection of the buy price (the treatment dummy is statistically significant with a p-value of 0.04).

Table 7(b) compares revenue in auctions in which the buy price is accepted to revenue in the identical ascending bid auctions. For each buy now auction session, revenue when the buy price is accepted ($5.60) exceeds average revenue across the identical ascending bid auction sessions (show in the right most column of the table).

Figure 6 shows observed dropout prices in the ascending bid auctions. Comparing dropout prices to the risk-neutral rational bidding function $b_0(x) = \frac{3}{4}x$, it is apparent that bidders overbid for low signals (i.e., they drop out too late) but underbid when they have high signals. The figure does not conclusively show underbidding for high signals since there is a selection bias – we don’t observe the highest dropout price in an auction.

Figure 6 goes here.

To whether determine whether (i) bidders underbid when they have high signals and (ii) bid differently following the rejection of the buy price, we estimated linear bid functions using a censored regression model. In each auction we observe the dropout prices of three bidders, but not the dropout price of the bidder who wins the auction. We assume that the dropout price of the $i$-th bidder, $i \in \{1, 2, 3, 4\}$, in auction $t$ is given by

$$b_i^t = \gamma + \beta x_i^t + \varepsilon_i^t,$$

where $x_i^t$ is the signal of bidder $i$ in auction $t$, and $\varepsilon_i^t$ is distributed according to $N(0, \sigma_2)$. The likelihood function has signals rather than values as the independent variable, but is otherwise identical to the one given earlier for private value ascending bid auctions.

Table 8 reports the results of maximum likelihood estimation of $\gamma$ and $\beta$. The first column shows the results using the data from the 180 ascending auctions, while the second column is based on the 38 ascending auctions after the buy price is rejected. Standard errors of the estimates are reported within parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Ascending</th>
<th>Buy-now ($B$ rejected)</th>
<th>z-test ($p$-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\gamma}$</td>
<td>2.153 (0.121)</td>
<td>1.774 (0.192)</td>
<td>0.08</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>0.436 (0.020)</td>
<td>0.335 (0.047)</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Obs. 180 38

Table 8: ML Estimated Bidding Functions
The estimated intercept ($\gamma$) is significant in both types of auctions. However, one can reject the null hypothesis that $\gamma$ is the same for both types of auctions ($p$-value of 0.08); bidders drop out earlier in an ascending auction following the rejection of the buy price, a finding consistent with Table 7. In contrast, the equilibrium (rational) bidding function is the same and has an intercept of zero in both types of auctions. One cannot reject that $\beta$ is the same for both types of auctions ($p$-value of 0.64); the rejection of the buy price has no statistically significant effect on the responsiveness of bids to signals.

A bidder’s decision to accept the buy price depends on his signal, as shown in the Probit regressions reported in Table 9.

<table>
<thead>
<tr>
<th></th>
<th>All Data</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Marginal</td>
<td>$p$-val.</td>
<td>Coefficient</td>
<td>Marginal</td>
<td>$p$-val.</td>
</tr>
<tr>
<td>Constant</td>
<td>-2.296 (0.148)</td>
<td>0.00</td>
<td></td>
<td>-2.908 (0.430)</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Signal $x_i$</td>
<td>0.325 (0.022)</td>
<td>0.105 (0.007)</td>
<td>0.00</td>
<td>0.400 (0.054)</td>
<td>0.154 (0.021)</td>
<td>0.00</td>
</tr>
<tr>
<td>Obs.</td>
<td>720</td>
<td></td>
<td></td>
<td>318</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9: Probit – Decision to Accept Buy Price in CV auctions.

The probability that a bidder accepts buy price is increasing in his signal, whether one considers either the overall data or just the data for bidders whose signals exceeded the buy price.

The results for common values auctions depart from the rational bidding theory in four significant ways: (i) The buy price is accepted too frequently. For the signals received by bidders in our experiment, the buy price should never be accepted if bidders are either risk neutral or risk averse. In fact it was accepted in 79% of all auctions. (ii) Subjects bid less in auctions where the buy price has been rejected than in identical auctions where no buy price is offered. (iii) Subjects overbid relative to theoretical predictions when they have low signals, but underbid when they have high signals.27 (iv) The buy price raises seller revenue rather than reducing it. In the next section we introduce a model which explains all these features of the data.

27The equilibrium rational bidding function has a zero intercept for any level of risk aversion, hence the rational bidding function does not explain the observed overbidding for low signals even if bidders are risk averse.
5 Naive Bidding in Buy-Now Auctions

In this section we introduce a behavioral model in order to explain the results from the common value buy-now auctions. We first focus on bidding behavior in the ascending auctions, and then consider the decision to accept or reject the buy price.

**Bidding in Common Value Auctions**

Kagel and Levin’s (1986) expected value (EV) bidding model is a simple model of the winner’s curse in common-value auctions. According to this model, when forming their bid, bidders fail to condition on their signal being highest when they win the auction. In an ascending bid auction, for example, they bid up to their expected value (when risk neutral) of the item conditional on their own signal. The EV bidding function, when bidders have index of risk aversion \(\alpha\), is denoted by \(b^E\alpha_{EV}(x)\) and satisfies

\[
E[u(v - b^E\alpha_{EV}(x)) | x_1 = x] = 0.
\]

We also consider the possibility that bidders may overweight their own signal when calculating the value of the item. We denote the EV bidding function augmented with overweighting of own signal by \(b^E\alpha_{EV}(x)\). It satisfies

\[
E\left[u\left(\frac{\lambda x_1 + x_2 + \cdots + x_n}{\lambda + n - 1} - b^E\alpha_{EV}(x)\right) | x_1 = x\right] = 0,
\]

where \(\lambda\) denotes the degree to which a bidder overweights (if \(\lambda > 1\)) or under weights (if \(\lambda < 1\)) his own signal. If \(\lambda = 1\) this model reduces to EV bidding.\(^{28}\)

In a buy-now auction, the ascending bid phase of the auction is only reached if all the bidders reject the buy price at the first stage. Hence, in the ascending bid phase of the auction a bidder will condition on all of his rivals having a signal below the equilibrium cutoff \(c\). A bidder who correctly conditions on his rivals’ signals being less than \(c\), but who fails to condition on his signal being highest when he wins, will bid according to \(b^E\alpha_{EV}(x, c)\) defined by

\[
E[u(v - b^E\alpha_{EV}(x, c)) | x_1 = x, z \leq c] = 0.
\]

If, in addition, the bidder incorrectly weights his own signal, then he bids according to \(b^E\alpha_{EV}(x, c)\) defined by

\[
E\left[u\left(\frac{\lambda x_1 + x_2 + \cdots + x_n}{\lambda + n - 1} - b^E\alpha_{EV}(x, c)\right) | x_1 = x, z \leq c\right] = 0.
\]

\(^{28}\)Goeree and Offerman (2002) consider a naive bidding model for first-price sealed-bid auctions in which bidders over or underweight their common value signal relative to their private value signal.
In contrast to the rational model, the EV and EV+ models both predict less aggressive bidding following the rejection of the buy price, so long as \( c < \bar{v} \) (\( \bar{v} \) = $10).

Table 10 shows the theoretical bidding functions for the three alternative bidding models (rational, EV, and EV+) for the parameterization of our experiment and, for comparison, it shows on the last row the estimated bid functions from Table 8. For simplicity, the table only provides the risk-neutral bidding functions, in which case all three bidding functions are linear. The rational bidding function has a zero intercept, while the EV and EV+ bidding functions both have positive intercepts and hence predict overbidding by bidders with low signals. However, if bidders employ a cutoff \( c < $10 \), then predicted overbidding is reduced in both models when the buy price is rejected. Bidders in the EV+ model overweight their own signals and hence the bidding function is steeper than in the EV model.

<table>
<thead>
<tr>
<th>Ascending</th>
<th>Buy-now (B rejected)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rational</td>
<td>( b_0(x) = \frac{3}{4}x )</td>
</tr>
<tr>
<td>EV</td>
<td>( b_0^{EV}(x) = \frac{3}{8}(10) + \frac{1}{4}x )</td>
</tr>
<tr>
<td>EV+</td>
<td>( b_0^{EV+}(x) = \frac{3}{2(\lambda+3)}(10) + \frac{\lambda}{\lambda+3}x )</td>
</tr>
<tr>
<td>Estimated</td>
<td>( b(x) = 2.153 + 0.436x )</td>
</tr>
</tbody>
</table>

Table 10: Risk-neutral Bidding Functions (cutoff of \( c \))

In both the ascending bid auctions without a buy price and the ascending bid auctions following the rejection of the buy price, the estimated intercepts are positive, which suggests that bidders suffer from the winner’s curse. The estimated slopes of 0.436 and 0.335 are greater than \( \frac{1}{4} \), which suggests that bidders overweight their own signals. In particular, the .436 estimate implies \( \lambda = 2.32 \) and \( \alpha = 1.617 \).\(^{29}\) Since the data suggests that bidders suffer from the winner’s curse and that they also overweight their own signals, henceforth we focus on the EV+ model.

\(^{29}\)When bidders have index of risk aversion \( \alpha \), then the equilibrium bid function in the EV+ model of the ascending auction is

\[
b^{EV+}_0(x) = -\frac{1}{\alpha} \ln \left( \int_0^{10} \int_0^{10} \int_0^{10} e^{-\alpha(\gamma_2(x_2+x_3+x_4))} \frac{1}{10^3} dx_2 dx_3 dx_4 \right) + \frac{\lambda}{\lambda+3}x.
\]

Since \( \beta = \lambda/(\lambda + 3) \), then \( \lambda \) can be inferred from the estimate of \( \beta \) and then, given \( \lambda \), we can infer \( \alpha \) from the estimated intercept.
**The Decision to Accept or Reject the Buy Price**

Next we investigate whether the winner’s curse and overweighing of own signal can explain the high frequency with which the buy price is accepted. A bidder who accepts the buy price wins with probability \( \frac{1}{1+n} \) if exactly \( l \) of his rivals have values above the cutoff \( c \). A rational bidder accounts for the fact that if he accepts the buy price, he is more likely to win as more of his rivals have values below \( c \). We suppose instead that bidders are also subject to the winner’s curse when accepting the buy price, with a bidder whose signal is \( x \) computing the expected utility to accepting the buy price as

\[
\hat{U}^A(x, c; \lambda) = E \left[ u \left( \frac{\lambda x_1 + x_2 + \cdots + x_n}{\lambda + n - 1} - B \right) \bigg| x_1 = x \right] Q(c),
\]

where \( Q(c) \) is the bidder’s probability of winning the item when all his rivals employ the cutoff \( c \). A bidder computes the expected utility of rejecting the buy price as

\[
\hat{U}^R(x, c; \lambda) = E \left[ u \left( \frac{\lambda x_1 + x_2 + \cdots + x_n}{\lambda + n - 1} - b^E_{\alpha}(z, c) \right) \bigg| x_1 = x, z \leq c \right] G(\min\{x, c\}).
\]

Such a bidder correctly calculates the probability of ultimately winning the auction if he rejects the buy price. He also conditions on all his rivals’ signals being less than \( c \) if no bidder accepts the buy price. However, consistent with the winner’s curse, he fails to condition on his signal being highest when bidding in the ascending bid auction.

A cutoff \( c^* \) is a **symmetric Bayes Nash equilibrium** if \( \hat{U}^R_{\alpha}(x, c^*; \lambda) > \hat{U}^A_{\alpha}(x, c^*; \lambda) \) for all \( x \in [\underline{x}, c^*] \) and \( \hat{U}^R_{\alpha}(x, c^*; \lambda) < \hat{U}^A_{\alpha}(x, c^*; \lambda) \) for all \( x \in (c^*, \overline{v}] \). We write \( c^*_{\alpha}(\lambda) \) for the equilibrium cutoff as a function of \( \lambda \) and \( \alpha \).

We use maximum likelihood techniques to estimate \( \alpha \) and \( \lambda \) from the buy-now auction data. In each auction we observe whether a bidder accepts the buy price. If no bidder accepts the buy price, then we observe the dropout price of each bidder (except the winning bidder) in the ascending bid auction. Define

\[
\Delta_{\alpha}(x; \lambda) = \hat{U}^A_{\alpha}(x, c^*_{\alpha}(\lambda); \lambda) - \hat{U}^R_{\alpha}(x, c^*_{\alpha}(\lambda); \lambda)
\]

as the difference between the payoff to accepting and rejecting the buy price when all of a bidder’s rivals employ the cutoff \( c^*_{\alpha}(\lambda) \). Let \( D^i_t \) be a dummy variable which equals 1 if bidder \( i \) in auction \( t \) accepts the buy price and which equals zero otherwise. The
econometric model underlying our estimation is

\[
D^t_i = \begin{cases} 
1 & \text{if } \Delta_\alpha(x^t_i; \lambda) \geq \varepsilon^t_i \\
0 & \text{if } \Delta_\alpha(x^t_i; \lambda) < \varepsilon^t_i,
\end{cases}
\]

where \(\varepsilon^t_i\) is \(N(0, \sigma^t_1)\). We assume that bidder \(i\)'s bid in auction \(t\) is given by

\[
b^t_i = b^{EV+}_\alpha(x^t_i, c^*_\alpha(\lambda)) + \eta^t_i,
\]

where \(\eta^t_i\) is distributed \(N(0, \sigma^t_2)\).\(^{30}\) Let \(k^t \in \{1, 2, 3, 4\}\) be the winner of auction \(t\), i.e., bidder \(k^t\) has the highest (but unobserved) dropout price, and let \(\bar{b}^t\) be the highest observed dropout price.

The likelihood function for auction \(t\) is given by

\[
L^t = \begin{cases} 
1 - \prod_{i=1}^{4} \left[ 1 - \Phi \left( \frac{\Delta_\alpha(x^t_i; \lambda)}{\sigma^t_1} \right) \right] & \text{if } D^t_i = 1 \text{ for some } i \\
\prod_{i=1}^{4} \left[ 1 - \Phi \left( \frac{\Delta_\alpha(x^t_i; \lambda)}{\sigma^t_1} \right) \right] \left[ 1 - \Phi \left( \frac{b^t_i - b^ {EV+}_\alpha(x^t_i, c^*_\alpha(\lambda))}{\sigma^t_2} \right) \right] \prod_{i \in \{1,2,3,4\} \setminus \{k^t\}} \frac{1}{\sigma^t_2} \phi \left( \frac{b^t_i - b^ {EV+}_\alpha(x^t_i, c^*_\alpha(\lambda))}{\sigma^t_2} \right) & \text{otherwise}
\end{cases}
\]

The first column of Table 11 reports the results of maximizing \(\hat{L}^t = \sum_{t=1}^{T} L^t\) with respect to \(\alpha\) and \(\lambda\) (and \(\sigma^t_1\) and \(\sigma^t_2\)) using the data from the buy-now auctions. The estimated values of \(\lambda\) and \(\alpha\) are 5.118 and 0.0007, which are both statistically significant, although the estimated degree of risk aversion is small. The equilibrium cutoff for these values of \(\lambda\) and \(\alpha\) is \(c^*_\alpha(\lambda) = 6.842\), which implies that the probability the buy price is accepted (by at least one bidder) is .78. Since the probability that the buy price is accepted is increasing in \(\lambda\) and decreasing in \(\alpha\), the high rate (79\%) with which the buy price is accepted implies a high value of \(\lambda\) and a low value of \(\alpha\) compared to the estimates obtained from the ascending bid auctions alone. (See, for comparison, the right hand side of Table 11 which reports the values of \(\lambda\) and \(\alpha\)

\[b^ {EV+}_\alpha(x^t_i, c^*_\alpha(\lambda)) = -\frac{1}{\alpha} \ln \left( \int_0^{c^*_\alpha(\lambda)} \int_0^{c^*_\alpha(\lambda)} \int_0^{c^*_\alpha(\lambda)} e^{-\gamma x^t_2 x^t_3 x^t_4} \frac{1}{c^*_\alpha(\lambda)^3} dx^t_2 dx^t_3 dx^t_4 \right) + \frac{\lambda}{3} x^t_1.\]
implied by the estimates in Table 8.)

<table>
<thead>
<tr>
<th></th>
<th>Buy-now</th>
<th>Buy-now + Ascending</th>
<th>Ascending</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\alpha}$</td>
<td>0.0007 (0.0004)</td>
<td>0.7488 (0.015)</td>
<td>1.617 (0.229)</td>
</tr>
<tr>
<td>$\hat{\lambda}$</td>
<td>5.1184 (0.0012)</td>
<td>3.7488 (0.001)</td>
<td>2.320 (0.191)</td>
</tr>
<tr>
<td>$c^*_a(\hat{\lambda})$</td>
<td>6.842 (0.000)</td>
<td>7.3921 (0.000)</td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>180</td>
<td>360</td>
<td>180</td>
</tr>
</tbody>
</table>

Table 11 - ML Estimates for EV+

The second column of Table 11 reports the estimates for the EV+ model based on both the buy-now and the ascending bid auctions. The estimated equilibrium cutoff in this case is $c^*_a(\lambda) = 7.3921$, which implies that the probability the buy price is accepted is .70.

We conclude this section by comparing the revenue predictions of the rational and EV+ models. Recall that under the rational model, in equilibrium the $5.60 buy price is always rejected and seller revenue is $4.50 if bidders are risk neutral (and less if bidders are risk averse). In contrast, in the data the buy price was taken in 79% of all auctions and mean seller revenue was $5.25. Based on the parameter estimates in column 1 of Table 11, in the equilibrium of the EV+ model the buy price is taken in 73% of the auctions (conditional on the signals drawn) and seller revenue is $5.19. Hence, while the EV+ model does not fully explain the high rate with which the buy price is accepted, it does largely explain the high revenue achieved by the buy-it-now auctions.

The EV+ model, however, does not explain the modest revenue advantage of the buy-now auction over the ascending auction found in the data, if one insists that $\alpha$ and $\lambda$ are stable across treatments. Using the estimates from the first and the second columns of Table 11, the predicted revenues in the ascending auctions (conditional on the signal draws) are $5.75$ and $5.46$, respectively, while predicted revenues in the buy-now auction are only $5.19$ and $5.08$. Even if the buy-now auction yields less revenue, a risk averse seller might prefer it since it reduces the variance of seller revenue. Another possibility is that, as our estimation results suggest, parameters are not stable across treatments. The third-column parameter estimates in Table 11 imply that revenue in the ascending auction is only $4.85$, which is less than $5.19$ predicted for the buy-now auction based on the first-column estimates.
6 Conclusion

When values are private, theory suggests that buy-now auctions are potentially advantageous to the seller as they raise more revenue (when bidders are risk averse) while reducing the variance of revenue. Our experimental results demonstrate that both these predictions are borne out in the data.

Theoretically there is no revenue advantage to a buy price when values are common – any buy price accepted with positive probability reduces the seller’s expected revenue. We find that employing a buy price reduces the variance of seller revenue (with the buy price frequently accepted by bidders) and yet it leads to no reduction in revenue. In other words, while a buy price does not “work” for sellers in theory, it does work in practice. It works since it exploits naive bidding and the failure of bidders to correctly weight their own signals. That buy prices “work,” both when values are private and when they are common, potentially explains why they are so widely used in practice.

Kagel and Levin’s naive bidding model has found strong support in standard common-value auctions, explaining overbidding by subjects with low signals and underbidding by subjects with high signals. Extended to buy-now auctions, the naive bidding model predicts that subjects bid less in ascending bid auctions following the rejection of the buy price, whereas the rational model predicts no change in bidding behavior. Our experimental results strongly support less-aggressive bidding following the rejection of the buy price, thereby providing support for the naive bidding model in a novel way.

References


Figure 1
Expected Seller Revenue as a Function of the Buy Price, $\alpha=0$ and $\alpha=1$
Figure 2
Expected Utility as a Function of the Bidder's Signal

$\hat{U}_0^R(x, 9.96)$

$\hat{U}_0^A(x, 9.96)$

$\hat{U}_1^R(x, 10.00)$

$\hat{U}_1^A(x, 10.00)$
Figure 3
Expected Seller Revenue as a Function of the Buy Price, $\alpha=0$ and $\alpha=1$
Figure 4

c.d.f. of Seller Revenue in Private-Value Auctions
With and Without a Buy Price of $8.10
Figure 5

c.d.f. of Seller Revenue in Common-Value Auctions
With and Without a Buy Price of $5.60

Revenue
Figure 6
Dropout Prices in Ascending Common-Value Auctions